Japanese Monetary Policy Reaction Function and Time-Varying Structural Vector Autoregressions: A Monte Carlo Particle Filtering Approach *

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Abstract

In recent years, Japanese monetary policy has been a hot topic when discussing Japan's economy. This paper presents empirical analysis of Japanese monetary policy based on time-varying structural vector autoregressions (TVSVAR). Our TVSVAR includes a monetary reaction function, an aggregate supply function, an aggregate demand function, and effective exchange rate determination function. Our TVSVAR is a dynamic full recursive structural VAR, which is similar to Primiceri (2005), Canova and Gambetti (2006), and many related papers. Most previous TVSVAR studies are based on Markov Chain Monte Carlo method and the Kalman filter. We, however, adopt a new TVSVAR estimation method, proposed by Yano (2008). The method is based on the Monte Carlo Particle filter and a self-organizing state space model, proposed by Kitagawa (1996), Gordon et al. (1993), Kitagawa (1998), Yano (2007b), and Yano (2007a). Our method is applied to the estimation of a quarterly model of the Japanese economy (a nominal short term interest rate, inflation rate, real growth rate, and nominal effective exchange rate). We would like to emphasize that our paper is the first to analyze the Japanese economy using TVSVAR. Our analysis indicates that the monetary policy of Japan becomes ineffective in 1990s. Whether the long-term recession experienced by Japan in the 1990s was caused by aggregate supply factors or aggregate demand factors is an oft heard question. This paper concludes that both supply and demand factors contributed to the 10-year recession.

Key words： Monte Carlo particle filter, Self-organizing state space model, time-varying-coefficient, structural vector autoregressions, Markov chain prior.

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1 Introduction

In recent years, Japanese monetary policy has been a hot topic when discussing Japan's economy. This paper presents empirical analysis of Japanese monetary policy using time-varying structural vector autoregressions (TVSVAR). Our TVSVAR includes a monetary reaction function, an aggregate supply function, an aggregate demand function, and effective exchange rate determination function. The changes in coefficients indicate changes in the correlations among macroeconomic variables. Thus, we are able to analyze changes in the Japanese economy. Our approach is related to Uhlig (1997), Cogley and Sargent (2001), Ciccarelli and Rebucci (2003), Cogley and Sargent (2005), Primiceri (2005), Sims and Zha (2006), Canova and Gambetti (2006), and many studies. Most previous studies are based on Markov Chain Monte Carlo method and the Kalman filter[4]. In the studies, the random walk prior (the Minnesota/Litterman prior), proposed by Doan et al. (1984), is assumed on the time-evolutions of coefficients. The prior is based on linear Gaussian state space modeling [5]. Yano (2008), however, proposes a new TVSVAR estimation method that is based on the Monte Carlo Particle filter and a self-organizing state space model, proposed by Kitagawa (1996), Gordon et al. (1993), Kitagawa (1998), Yano (2007b), and Yano (2007a). A novel feature of Yano (2008) is that it assumes the time evolutions of coefficients are given by Markov chain processes. We call this assumption the Markov chain prior on time-varying coefficients. Our prior is based on nonlinear non-Gaussian state space modeling. The linear Gaussian case of the Markov chain prior is equivalent to the random walk prior. Thus, our method is more flexible rather than previous methods. Our method is applied for the estimation of a quarterly model of the Japanese economy (a nominal short-term interest rate, inflation rate, real growth rate, and nominal effective exchange rate).

There exist previous studies on Japanese monetary policy based on Bayesian statistical approach: Kimura et al. (2003), Fujiwara (2006), and Inoue and Okimoto (2007) [6]. Kimura et al. (2003) estimates time-varying reduced-form VAR models based on the Kalman filter. Fujiwara (2006) and Inoue and Okimoto (2007) analyze regime changes in the Japanese economy in the 1990s using Markov switching VAR (MSVAR). The main advantages of our method to the previous studies are that we need fewer restrictions on the time-evolution of coefficients and less prior knowledge of structural changes. Kimura et al. (2003) assume the random walk prior (the Minnesota prior) on the time-evolution of coefficients, which are based on linear Gaussian state space modeling. We, however, adopt the Markov chain prior, which assumes that the time-evolutions of coefficients follow Markov chain processes. Our assumption is less restricted rather the random walk prior. Fujiwara (2006) and Inoue and Okimoto (2007), use prior knowledge of the number of structural changes in the Japanese economy. In our method, the structural changes of coefficients of the economy are detected using the estimated time-varying coefficients of our model. Thus, we do not need prior knowledge of the structural changes of coefficients and regime changes in the Japanese economy. Moreover, an advantage of our approach to the MSVAR approach is that one can use a whole data set to estimate TVSVAR, even if there are several structural changes.

The major findings of this paper are summarized as follows. (i) The Bank of Japan's conduct of monetary policy by changing interest rates worked well to control real GDP in the 1980s. However, it has not worked to control real GDP since the 1990s. Furthermore, lower interest rates brought lower economic growth. (ii) The interest rate has had almost no impact on the inflation rate since 1990 even though interest rate policy worked to control inflation in the 1980s. (iii) Policy reaction of the interest rate to the inflation rate was strong in the early 1980s. However, interest rate reaction to the inflation rate diminished dramatically after 1997. In particular,

[4]Canova (2007) and Dejong and Dave (2007) are introductory textbooks on the Bayesian statistical approach for macroeconomic analysis. Fernandez-Villaverde and Rubio-Ramirez (2005) and Fernandez-Villaverde and Rubio-Ramirez (2007) have shown that the Monte Carlo particle filter and maximizing likelihood can be successfully applied to estimate DSGE models.

[5]The random walk prior is equivalent to the first-order of the smoothness prior, proposed by Kitagawa (1983). TVSVAR based on the Kalman filter is adopted in Jiang and Kitagawa (1993) and Yano (2004) to estimate reduced-form time-varying-coefficient vector autoregressions.

[6]Miyao (2006) is a comprehensive survey of the Japanese macroeconomic and monetary policy based on structural VARs. Kasuya and Tanemura (2000) constructs Bayesian VAR optimized by the Posterior Information Criterion and estimates the performance of forecasting.

introduction of the zero interest rate policy bounded by zero meant the central bank of Japan could not set interest rates in negative territory. (iv) Whether the long-term recession experienced by Japan in the 1990s was caused by aggregate supply factors (Hayashi and Prescott (2002), Hayashi (2003) and Miyao (2006)) or aggregate demand factors (such papers as Kuttner and Posen (2001) and Kuttner and Posen (2002)) is an oft heard question. This paper will show both supply and demand factors contributed to the 10-year recession. (v) From aggregate demand estimates, we can find spiral effects in the Japanese economy, especially after 1995. Lower real GDP and a lower inflation rate exacerbated the sluggish economy creating a downward spiral. (vi) From aggregate supply estimates, the spiral effects are not observed in the sense that lower inflation did not cause much of a further decline in the inflation rate.

This paper is organized as follows. In section 2, we describe time-varying structural autoregressions and the outline of a new TVSVAR estimation method, proposed by Yano (2008). In section 3, we give empirical analyses of Japanese monetary policy and the Japanese economy. In section 4, we give conclusions and some discussion.

2 Time-Varying Structural Vector Autoregressions

In this section, we give an outline of Yano (2008). First, we describe time-varying structural vector autoregressions and define state vectors to estimate them. Second, we explain the Monte Carlo particle filter and a self-organizing state space model to estimate a non-linear non-Gaussian state space model.

2.1 Time-Varying Structural Vector Autoregressions

Time-varying structural vector autoregressions (TVSVAR) for the time series $Y_{1:T} = \{Y_1, Y_2, \cdots, Y_T\}$ are defined as follows:

$$B_{0,t}Y_t = \sum_{p=1}^{P} B_{p,t}Y_{t-p} + D_t u_{t-\kappa} + c_t + \epsilon_t, \tag{1}$$

where Y_t is a $(k \times 1)$ vector of observations at time t, $u_{t-\kappa}$ is an $(n \times 1)$ vector of exogenous variables at time t, $\kappa \geq 0$ is a constant, c_t is a $(k \times 1)$ vector of time-varying intercepts at time t, and the error vector with stochastic volatility, $\epsilon_t = (\epsilon_{1,t}, \cdots, \epsilon_{k,t})^T \sim N(0, V_t)$ with $V_t = \mathrm{diag}(\sigma^2_{1,t}, \sigma^2_{2,t}, \cdots, \sigma^2_{k,t})$ [7]. The matrices of time varying coefficients are:

$$B_{0,t} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -b_{2,1,0,t} & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -b_{k,1,0,t} & \cdots & -b_{k,k-1,0,t} & 1 \end{bmatrix}, \tag{2}$$

$$B_{p,t} = \begin{bmatrix} b_{1,1,p,t} & \cdots & b_{1,k,p,t} \\ \vdots & \ddots & \vdots \\ b_{k,1,p,t} & \cdots & b_{k,k,p,t} \end{bmatrix}, \tag{3}$$

and

$$D_t = \begin{bmatrix} d_{1,1,t} & \cdots & d_{1,n,t} \\ \vdots & \ddots & \vdots \\ d_{k,1,t} & \cdots & d_{k,n,t} \end{bmatrix}. \tag{4}$$

Our TVSVAR is a generalization of TVSVAR, proposed by Primiceri (2005), because we add the exogenous vector, $u_{t-\kappa}$. We would like to stress that Eq. (1) is a general formulation of time-varying-coefficient regression/autoregression modeling. Jiang and Kitagawa (1993) pointed out that Eq. (1) can be estimated by each

[7] In this paper, a bold-faced symbol means a vector or a matrix.

component of Y_t because V is a diagonal matrix. For example, $Y_{1,t}$, the first component of Y_t in Eq. (1), can be written by

$$\begin{aligned} Y_{1,t} = & b_{1,1,1,t}Y_{1,t-1} + b_{1,2,1,t}Y_{2,t-1} \cdots + b_{1,k,1,t}Y_{k,t-1} + \\ & \cdots + b_{1,1,p,t}Y_{1,t-1} + b_{1,2,p,t}Y_{2,t-1} \cdots + b_{1,k,p,t}Y_{k,t-1} + c_{1,t} + \epsilon_{1,t}, \end{aligned} \tag{5}$$

where $c_{1,t}$ is the first component of c_t, and $\epsilon_{1,t}$ is the first component of $\boldsymbol{\epsilon}_t$. For another example, $Y_{2,t}$, the second component of Y_t in Eq. (1), can be written by

$$\begin{aligned} Y_{2,t} = & b_{2,1,0,t}Y_{1,t} + b_{2,1,1,t}Y_{1,t-1} + b_{2,2,1,t}Y_{2,t-1} \cdots + b_{2,k,1,t}Y_{k,t-1} + \\ & \cdots + b_{2,1,p,t}Y_{1,t-1} + b_{2,2,p,t}Y_{2,t-1} \cdots + b_{2,k,p,t}Y_{k,t-1} + c_{2,t} + \epsilon_{2,t}, \end{aligned} \tag{6}$$

where $c_{2,t}$ is the second component of c_t, and $\epsilon_{2,t}$ is the second component of $\boldsymbol{\epsilon}_t$. For the first example, we define a state vector x_t of time varying coefficients as follows:

$$x_t = \left[b_{1,1,1,t}, b_{1,2,1,t}, \cdots, b_{1,k,1,t}, \cdots, b_{1,1,p,t}, b_{1,2,p,t}, \cdots b_{1,k,p,t}, c_{1,t} \right]^T. \tag{7}$$

For the second example, we define another state vector x_t of time varying coefficients as follows:

$$x_t = \left[b_{2,1,0,t}, b_{2,1,1,t}, b_{2,2,1,t}, \cdots, b_{2,k,1,t}, \cdots, b_{2,1,p,t}, b_{2,2,p,t}, \cdots b_{2,k,p,t}, c_{2,t} \right]^T. \tag{8}$$

Note that in the two examples above, we ignore the exogenous vector, $u_{t-\kappa}$. Generalized formulations are described in Appendix D. In time-varying-coefficient regression/autoregression modeling, the main problem is how to estimate the state vector x_t [8]. In the framework of sequential Bayesian filtering, the filtering distribution of x_t, which is based on the observation vector, $Y_{1:t}$, is given by

$$p(x_t|Y_{1:t}). \tag{9}$$

The smoothing distribution of x_t, which is based on the observation vector, $Y_{1:T}$, is given by

$$p(x_t|Y_{1:T}). \tag{10}$$

Moreover, we assume that the time evolution of x_t is given by

$$p(x_t|x_{t-1}). \tag{11}$$

We refer to this assumption as the Markov chain prior on time-varying coefficients. Our prior is based on nonlinear non-Gaussian state space modeling. The linear Gaussian case of the Markov chain prior is equivalent to the random walk prior, which has often been adopted in previous studies. We would like to emphasize that our Markov chain prior overcame the restriction of the random walk prior. Our problem is how to estimate the state vector x_t using Eq. (9), (10), and (11). To solve the problem, we adopt the Monte Carlo particle filter, which is an algorithm to estimate the state vector of a nonlinear non-Gaussian state space model. In the next subsection, we describe a method to estimate the state vector x_t using the filter.

2.2 Nonlinear Non-Gaussian State Space Modeling and Self-Organizing State Space Model

To estimate a state vector x_t, we adopt the Monte Carlo particle filter (MCPF), proposed by Kitagawa (1996), and Gordon et al. (1993) and a self-organizing state space model, proposed by Kitagawa (1998). In this subsection, we describe a nonlinear non-Gaussian state space model and a self-organizing state space model (MCPF is described in the next subsection).

[8] See Kitagawa and Gersch (1996).

A nonlinear non-Gaussian state space model for the time series $Y_t,\ t = \{1, 2, \cdots, T\}$ is defined as follows:

$$\begin{aligned} x_t &= f(x_{t-1} + v_t), \\ Y_t &= h_t(x_t + \epsilon_t), \end{aligned} \tag{12}$$

where x_t is an unknown $n_x \times 1$ state vector, v_t is $n_v \times 1$ system noise vector with a density function $q(v|\cdot)$ [9], and ϵ_t is $n_\epsilon \times 1$ observation noise vector with a density function $r(\epsilon|\cdot)$. The function $f : R^{n_x} \times R^{n_v} \to R^{n_x}$ is a possibly nonlinear function and the function $h_t : R^{n_x} \times R^{n_\epsilon} \to R^{n_y}$ is a possibly nonlinear time-varying function. The first equation of (12) is called a system equation and the second equation an observation equation. A system equation depends on a possibly unknown $n_s \times 1$ parameter vector, $\boldsymbol{\xi}_s$, and an observation equation depends on a possibly unknown $n_o \times 1$ parameter vector, $\boldsymbol{\xi}_o$. This nonlinear non-Gaussian state space model specifies the two following conditional density functions.

$$\begin{aligned} &p(x_t|x_{t-1}, \boldsymbol{\xi}_s), \\ &p(Y_t|x_t, \boldsymbol{\xi}_o). \end{aligned} \tag{13}$$

Note that $p(x_t|x_{t-1}, \boldsymbol{\xi}_s)$ is equivalent to Eq. (11). We define a parameter vector $\boldsymbol{\theta}$ as follows:

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\xi}_s \\ \boldsymbol{\xi}_o \end{bmatrix}. \tag{14}$$

We denote that θ_j is the j th element of $\boldsymbol{\theta}$ and $J (= n_s + n_o)$ is the number of elements of $\boldsymbol{\theta}$. This type of state space model (12) contains a broad class of linear, nonlinear, Gaussian, or non-Gaussian time series models. In state space modeling, estimating state vector, x_t, is the most important problem. For the linear Gaussian state space model, the Kalman filter, which is proposed by Kalman (1960), is the most popular algorithm to estimate state vector, x_t. For nonlinear or non-Gaussian state space models, there are many algorithms. For example, the extended Kalman filter (Jazwinski (1970)) is the most popular algorithm and the other examples are the Gaussian-sum filter (Alspach and Sorenson (1972)), the dynamic generalized model (West et al. (1985)), and the non-Gaussian filter and smoother (Kitagawa (1987)). In recent years, MCPF for nonlinear non-Gaussian state space models is a popular algorithm because it is easily applicable to various time series models [10].

In econometric analysis, generally, we do not know the parameter vector $\boldsymbol{\theta}$. In the TVSVAR framework, the unknown parameter vectors are $\boldsymbol{\xi}_o$ and $\boldsymbol{\xi}_s$. In traditional parameter estimation, maximizing the log-likelihood function of $\boldsymbol{\theta}$ is often used. The log-likelihood of $\boldsymbol{\theta}$ in MCPF is proposed by Kitagawa (1996). However, MCPF is problematic to estimate parameter vector $\boldsymbol{\theta}$ because of the likelihood of the filter containing errors from the Monte Carlo method. Thus, one cannot use a nonlinear optimizing algorithm like Newton's method [11]. To solve the problem, Kitagawa (1998) proposes a self-organizing state space model. In Kitagawa (1998), an augmented state vector is defined as follows:

$$z_t = \begin{bmatrix} x_t \\ \boldsymbol{\theta} \end{bmatrix}. \tag{15}$$

An augmented system equation and an augmented measurement equation are defined as

$$\begin{aligned} z_t &= F(z_{t-1}, v_t, \boldsymbol{\xi}_s), \\ Y_t &= H_t(z_t, \epsilon_t, \boldsymbol{\xi}_o), \end{aligned} \tag{16}$$

where

$$F(z_{t-1}, v_t, \boldsymbol{\xi}_s) = \begin{bmatrix} f(x_{t-1} + v_t), \\ \boldsymbol{\theta} \end{bmatrix}$$

[9]The system noise vector is independent of past states and current states.

[10]Many applications are shown in Doucet et al. (2001).

[11]See Yano (2007b).

and

$$H_t(z_t, \epsilon_t, \xi_o) = h_t(x_t + \epsilon_t).$$

This nonlinear non-Gaussian state space model is called a self-organizing state space (SOSS) model. This self-organizing state space model specifies the two following conditional density functions:

$$\begin{aligned} &p(z_t|z_{t-1}), \\ &p(Y_t|z_t). \end{aligned} \tag{17}$$

2.3 The Monte Carlo Particle Filter

Most algorithms of sequential Bayesian filtering are based on Bayes' theorem (see Arulampalam et al. (2002)), which is

$$P(z_t|Y_{1:t}) = \frac{P(Y_t|z_t)P(z_t|Y_{1:(t-1)})}{P(Y_t|Y_{1:(t-1)})}, \ t \geq 1, \tag{18}$$

where $P(z_t|Y_{1:t-1})$ is the prior probability, $P(Y_t|z_t)$ is the likelihood, $P(z_t|Y_{1:t})$ is the posterior probability, and $P(Y_t|Y_{1:t-1})$ is the normalizing constant. We denote an initial probability $P(z_0) = P(z_0|\emptyset)$, where the empty set $\emptyset$ indicates that we have no observations. In the state estimation problem, determining an initial probability $P(z_0)$, which is called filter initialization, is important because a proper initial probability improves a posterior probability. In TVSVAR, an initial probability is restricted in $-1 < x_{i,0} < 1$, where $x_{i,0}$ is the ith element of x_0.

In MCPF, the posterior density distribution at time t is approximated as

$$p(z_t|Y_{1:t}) = \frac{1}{\sum_{m=1}^{M} w_t^m} \sum_{m=1}^{M} w_t^m \delta(z_t - z_t^m), \tag{19}$$

where w_t^m is the weight of a particle z_t^m, M is the number of particles, and δ is Dirac's delta function [12]. The definition of w_t^m is described below. In the standard algorithm of MCPF, particles are resampled with sampling probabilities proportional to the weights w_t^m at every time t. It is necessary to prevent increasing the variance of weights after few iterations of Eq. (18) [13]. After resampling, the weights are reset to $w_t^m = 1/M$. Therefore, Eq. (19) is rewritten as

$$p(z_t|Y_{1:t}) = \frac{1}{M} \sum_{m=1}^{M} \delta(z_t - \hat{z}_t^m) \tag{20}$$

where $\hat{z}_t^m$ are particles after resampling. Using Eq. (20), the predictor $p(z_t|Y_{1:(t-1)})$ can be approximated by

$$\begin{aligned} p(z_t|Y_{1:(t-1)}) &= \int p(z_t|z_{t-1})p(z_{t-1}|Y_{1:(t-1)})dz_{t-1} \\ &= \frac{1}{M} \sum_{m=1}^{M} \int p(z_t|z_{t-1})\delta(z_{t-1} - \hat{z}_{t-1}^m)dz_{t-1} \\ &= \frac{1}{M} \sum_{m=1}^{M} p(z_t|\hat{z}_{t-1}^m) \\ &\simeq \frac{1}{M} \sum_{m=1}^{M} \delta(z_t - z_t^m). \end{aligned} \tag{21}$$

[12] The Dirac delta function is defined as

$$\begin{aligned} &\delta(x) = 0, \text{ if } x \neq 0, \\ &\int_{-\infty}^{\infty} \delta(x)dx = 1. \end{aligned}$$

[13] See Doucet et al. (2000).

Note that z_t^m is obtained from

$$z_t^m \sim p(z_t|\hat{z}_{t-1}^m). \tag{22}$$

Substituting Eq. (21) into Eq. (18), we obtain the following equation:

$$\begin{aligned} p(z_t|Y_{1:t}) &\propto p(Y_t|z_t)p(z_t|Y_{1:(t-1)}) \\ &\propto \frac{1}{M}p(Y_t|z_t)\sum_{m=1}^{M}\delta(z_t - z_t^m) \\ &= \frac{1}{M}\sum_{m=1}^{M}p(Y_t|z_t^m)\delta(z_t - z_t^m). \end{aligned} \tag{23}$$

Comparing Eq. (19) and Eq. (23) indicates that a weight w_t^m is obtain by

$$w_t^m \propto p(Y_t|z_t^m). \tag{24}$$

Therefore, a weight w_t^m is defined as

$$w_t^m \propto p(Y_t|z_t^m) = r(\psi_t(Y_t; z_t^m))\left|\frac{\partial\psi}{\partial y}\right|; m = \{1, \cdots, M\}, \tag{25}$$

where ψ_t is the inverse function of the function h_t [14]. In our TVSVAR estimation method, the augmented state vector is estimated using MCPF. Thus, states and parameters are estimated simultaneously without maximizing the log-likelihood of Eq. (16) because parameter vector $\boldsymbol{\theta}$ in Eq. (16) is approximated by particles and is estimated as the state vector in Eq. (15) [15]. The algorithm of our TVSVAR estimation method is summarized as follows [16].

Algorithm: Time-Varying Structural Vector Autoregressions Estimation

SOSS[$\{\hat{z}_{t-1}^m\}_{m=1}^M; y_t$]

{

FOR m=1,...M

Predict: $z_t^m \sim p(z_t|\hat{z}_{t-1}^m; v_t^m)$

Weight: w_t^m is obtained by Eq. (25)

ENDFOR

Sum of Weights: $sw = \sum_{m=1}^M w_t^i$

Log-Likelihood: $llk = \log(sw/M)$

FOR m=1,...,M

Normalize: $\hat{w}_t^m = \frac{w_t^m}{sw}$

ENDFOR

Resampling: [$\{\hat{z}_t^m; \hat{w}_t^m\}_{m=1}^M$] =resample[$\{z_t^m; \hat{w}_t^m\}_{m=1}^M$]

RETURN[$\{\hat{z}_t^m; \hat{w}_t^m\}_{m=1}^M$, llk]

}

SOSS.MAIN[$\{x_0^m\}_{m=1}^M; \{y\}_{t=1}^T$, P]

{

$\boldsymbol{\theta}_0 \sim uniform(P - r, P + r)$

$\{z_0^m\}_{m=1}^M = (\{x_0^m\}_{m=1}^M, \{\theta_0^m\}_{m=1}^M)$

FOR t=1,...,T

soss = SOSS[$\{\hat{z}_{t-1}^m\}_{m=1}^M; y_t$]

[14] See Kitagawa (1996).

[15] The justification of an SOSS model is described in Kitagawa (1998).

[16] The details of MCPF and SOSS are described in Yano (2007b).

$\{\hat{z}_t^m; \hat{w}_t^m\}_{m=1}^M = (\{\hat{z}_t^m; \hat{w}_t^m\}_{m=1}^M$ in soss)
ENDFOR
RETURN$[\{\{\hat{z}_t^m; \hat{w}_t^m\}_{m=1}^M\}_{t=1}^T, Y]$
}

With respect to a self-organizing state space model, however, Hürseler and Künsch (2001) points out a problem, namely determination of initial distributions of parameters for a self-organizing state space model. The estimated parameters of a self-organizing state space model comprise a subset of the initial distributions of parameters. We must know the posterior distributions of parameters to estimate parameters adequately. However, the posterior distributions of the parameters are generally unknown. Parameter estimation fails if we do not appropriately know their initial distributions. Yano (2007b) proposes a method to seek initial distributions of parameters for a self-organizing state space model using the simplex Nelder-Mead algorithm to solve the problem. To seek initial distributions of parameters, we adopt the algorithm, proposed by Yano (2007b). Moreover, we adopt the smoothing algorithm and filter initialization method, which is proposed by Yano (2007a).

2.3.1 Functional Forms

In this paper, we use linear non-Gaussian state space models to estimate time-varying coefficients and parameters. A linear non-Gaussian state is given by

$$\begin{aligned} x_t &= x_{t-1} + v_t, \\ Y_{i,t} &= H_t x_t + \epsilon_{i,t}, \end{aligned} \tag{26}$$

where $Y_{i,t}$ is an observation, $v_t \sim q(v_t|\boldsymbol{\xi}_t)$, $\epsilon_{i,t} \sim r_i(\epsilon_{i,t}|\xi_{i,o})$, $\epsilon_{i,t}$ is the ith component of $\boldsymbol{\epsilon}_t$, and $\xi_{i,o}$ is the ith component of $\boldsymbol{\xi}_o$. The details of x_t and H_t are described in Appendix C. In our Markov chain prior, $q(v_t|\boldsymbol{\xi}_t)$ and $r_i(\epsilon_{i,t}|\xi_{i,o})$ are possibly non-Gaussian distributions. We would like to emphasize that our prior make the estimation of TVSVAR flexible rather than the random walk prior. In this paper, the innovation term $q(v_t|\boldsymbol{\xi}_t)$ is specified by t-distributions, and $r_i(\epsilon_{i,t}|\xi_{i,o})$ is specified by a normal distribution. In general, the components, $\{\xi_{1,s}, \xi_{2,s}, \cdots, \xi_{L,s}\}$, of $\boldsymbol{\xi}_s$ are different (L is defined in Appendix C). In this paper, however, to reduce computational complexity, we assume as follows.

$$\xi_{1,s} = \xi_{2,s} = \cdots = \xi_{L,s} = |\xi_s| \tag{27}$$

In this paper, the time evolutions of coefficients are given by

$$x_{i,t} = x_{i,t-1} + |\xi_s| \times t(df), \tag{28}$$

where df is the degree of freedom of Student's t-distribution. The innovation term of Y_i is given by the normal distribution ($\epsilon_{i,t} \sim N(0, \sigma_{i,t}^2)$). A time-varying standard deviation is given by

$$\sigma_{i,t} = |\sigma_{i,t-1} + \xi_{i,o}\eta_t|, \tag{29}$$

where $\eta_t \sim N(0, 1)$.

3 Empirical Analyses

Our methods are applied for the estimation of a quarterly model of the Japanese economy. In the model, four variables are included : a short-term interest rate (the uncollateralized overnight call rate), inflation rate, growth rate, and nominal effective exchange rate [17]. We use data from 1980:Q1 up to 2006:Q3. The transformation of variables are (1) rate hikes, (2) growth rate of the seasonally-adjusted GDP deflator, (3) growth rate of seasonally-adjusted real GDP, and (4) the change of the nominal effective exchange rate. Rate hikes are given by the first

[17] Data details are described in Appendix A.

difference of the mean of the monthly average of the uncollateralized overnight call rate [18]. Growth rates of the GDP deflator and real output are given by

$$x_t = \left[\log X_t - \log X_{t-1} \right] \times 100. \tag{30}$$

The change of the nominal effective exchange rate is given by

$$e_t = -\left[\log E_t - \log E_{t-1} \right] \times 100, \tag{31}$$

where E_t is the nominal effective exchange rate. Note that e_t becomes smaller when the yen appreciates. In Fig. 1, the four variables are shown.

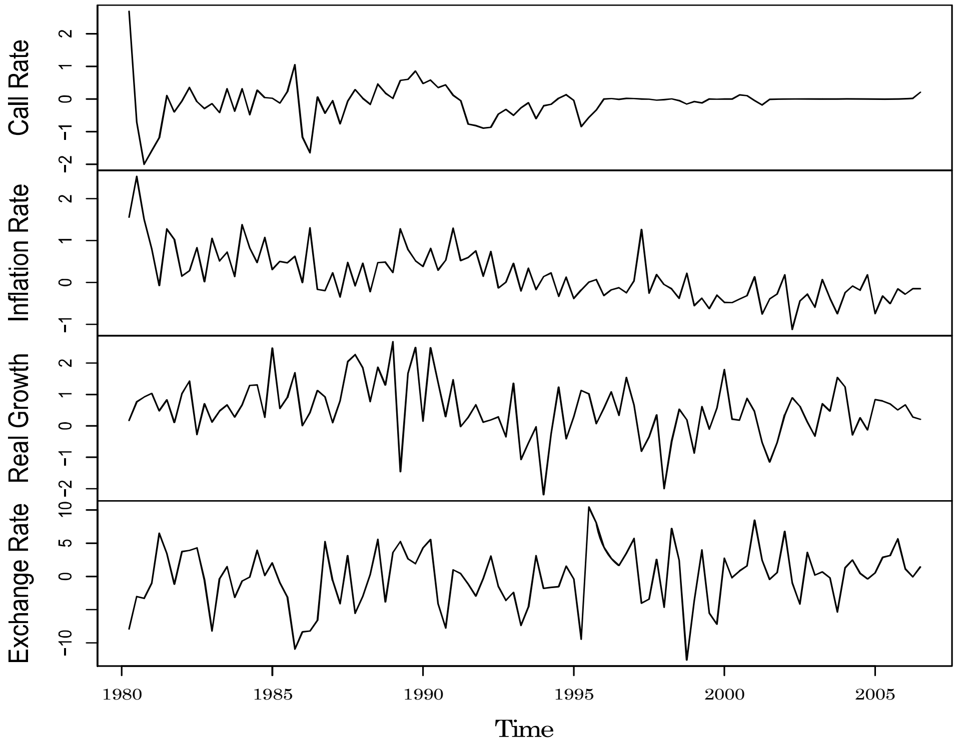

Figure 1: Macroeconomic data of Japan: 1980:Q1-2006:Q4

[18]See Miyao (2000), Miyao (2002), and Miyao (2006).

3.1 Full Recursive TVSVAR

We estimate the first order of full recursive TVSVAR (FR-TVSVAR(1)) as a benchmark model. FR-TVSVAR(1) is given by

$$i_t = b_{1,1,1,t} i_{t-1} + b_{1,2,1,t} \pi_{t-1} + b_{1,3,1,t} y_{t-1} + b_{1,4,1,t} e_{t-1} + c_{1,t} + \epsilon_{i,t}, \tag{32}$$

$$\pi_t = b_{2,1,0,t} i_t + b_{2,1,1,t} i_{t-1} + b_{2,2,1,t} \pi_{t-1} + b_{2,3,1,t} y_{t-1} + b_{2,4,1,t} e_{t-1} + c_{2,t} + \epsilon_{\pi,t}, \tag{33}$$

$$y_t = b_{3,1,0,t} i_t + b_{3,2,0,t} \pi_t + b_{3,1,1,t} i_{t-1} + b_{3,2,1,t} \pi_{t-1} + b_{3,3,1,t} y_{t-1} + b_{3,4,1,t} e_{t-1} + c_{3,t} + \epsilon_{y,t}, \tag{34}$$

$$e_t = b_{4,1,0,t} i_t + b_{4,2,0,t} \pi_t + b_{4,3,0,t} y_t + b_{4,1,1,t} i_{t-1} + b_{4,2,1,t} \pi_{t-1} + b_{4,3,1,t} y_{t-1} + b_{4,4,1,t} e_{t-1} + c_{4,t} + \epsilon_{e,t}. \tag{35}$$

where i_t is the first difference of the short-term interest rate, π_t is the inflation rate, y_t is the growth rate of real output, and e_t is the change of the nominal effective exchange rate [19]. Following Miyao (2000), Miyao (2002), and Miyao (2006), the variables of FR-TVSVAR(1) are ordered from exogenous variables to endogenous variables.

We estimate TVSVAR based on quarterly data of the Japanese economy from 1980:Q1 to 2006:Q3. Fig. 2, 3, 4, and 5 show Eq. (32), (33), (34), and (35), respectively. In these figures, the solid line is an estimate of a time-varying coefficient and the dashed lines are 68% confidence intervals [20]. In invariant-coefficient structural vector autoregressions (SVAR), impulse response functions (IRFs) are use to analyze the results. In our TVSVAR, IRFs are calculated in the same way in SVAR. However, the interpretation of IRFs of TVSVAR are different from the interpretation of IRFs of SVAR because the coefficients of TVSVAR are time-varying. In Yano and Yoshino (2008), we show IRFs for reference [21].

We compare TVSVAR with (invariant coefficient) Structural VAR (SVAR) using residual analysis. SVAR(P) is given by

$$B_0 Y_t = B_1 Y_{t-1} + B_2 Y_{t-2} + \cdots + B_P Y_{t-P} + \boldsymbol{\epsilon}_t$$

We estimate SVAR(1) using quarterly data of the Japanese economy from 1980:Q1 to 2006:Q3. B_0 of SVAR(1) is

$$B_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0490 & 1 & 0 & 0 \\ -0.4947 & 0.0100 & 1 & 0 \\ -0.7272 & -0.165 & -0.7892 & 1 \end{bmatrix}. \tag{36}$$

The standard error of B_0 is

$$B_0^{SE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2481 & 1 & 0 & 0 \\ 0.2483 & 0.1910 & 1 & 0 \\ 0.2548 & 0.1910 & 0.1157 & 1 \end{bmatrix}. \tag{37}$$

Eq. (37) shows that most of the standard errors in B_0 are larger than the elements of B_0. It indicates the estimates of SVAR (1) are unreliable. In Table 1, we show the root mean square error of FR-TVSVAR(1) and SVAR(1). It indicates that TVSVAR is better than SVAR.

In Fig 7 and 8, we show a quantile-quantile plot and the autocorrelation of residuals of FR-TVSVR(1), respectively. In Fig 9 and 10, we show a quantile-quantile plot and the autocorrelation of residuals of SVAR(1),

[19] We set the number of particles, M, to 10000. Moreover, we set the degrees of freedom, df, in Eq. (28) to 10, 20, 30. In the all cases, we get the same results. In our paper, we show results when we set df to 10. The other parameters of simulation are the same in Yano (2007b) and Yano (2007a). All time-varying coefficients are standardized as follows:

$$b_{x,y,z,t} = sd_{exp}/sd_{obs},$$

where sd_{exp} is the standard deviation of an explaining variable and sd_{exp} is the standard deviation of an observation (this standardization method may not be best). Eq. (32) is based on the Henderson-McKibbin-Taylor rule (see Clarida et al. (2000)).

[20] Confidence interval is calculated using 100 times estimation of a time-varying coefficient.

[21] Canova and Gambetti (2006) proposes to use generalized impulse response functions of TVSVAR to solve this problem.

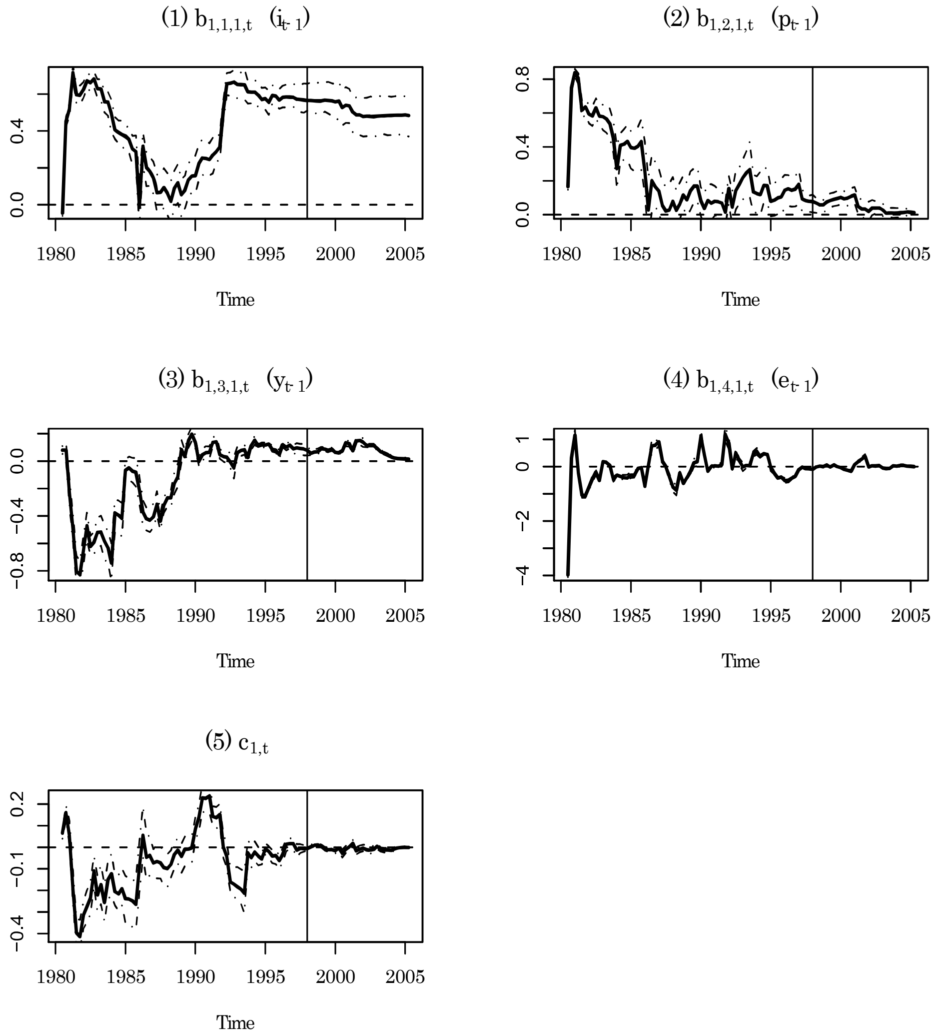

Figure 2: $i_t = b_{1;1;1;t} i_{t-1} + b_{1;2;1;t} \pi_{t-1} + b_{1;3;1;t} y_{t-1} + b_{1;4;1;t} e_{t-1} + c_{1;t} + \epsilon_{1;t}$

respectively. These results indicate that TVSVAR is better than SVAR.

3.2 Macroeconomic Analysis

In Fig. 2, the results of Eq. (32) are shown. Equation (32) represents the monetary policy reaction of the central bank of Japan. The call lending rate, which is controlled by the central bank of Japan, is assumed to depend on the following four variables, namely, (i) lagged interest rate which represents the smooth adjustment of the interest rate, (ii) inflation rate, (iii) growth rate of real GDP, and (iv) the exchange rate. Target values of the inflation rate, log of GDP, and the exchange rate are captured in changes in the constant term. Figures 2 (1) to 2 (5) show changes in the value of the coefficients of Equation (32). Figure 2 (1) shows that the central bank of Japan was conducting gradual adjustment of the short-term interest rate during 1980 to 1983 when a high inflation rate during the second oil crisis was the case. However, the central bank halted gradual interest rate adjustments during the bubble period (1984-90), though it resumed such adjustments from 1993, when the low (or zero) interest rate policy was adopted until recently. During the bubble period, the central bank's call rate control can be seen as abnormal compared

Table 1: Root Mean Square Error

RMSE	Time-varying SVAR	Invariant-coefficient SVAR
ϵ_i	0.04303	0.45066
ϵ_π	0.11014	0.52432
ϵ_y	0.08479	0.89801
ϵ_e	0.08479	4.15124

with other periods. Figure 2 (2) shows changes in the coefficient of the lagged rate of inflation. It was positive and quite significant between 1980 to 1982 when high inflation hit Japan. It indicates that the central bank was watching the rate of inflation as the major target of monetary policy. However, it turned to a negative value during the bubble period which suggests the central bank lowered its call lending rate despite a low rate of inflation during the asset price bubble period. Figure 2 (3) shows the monetary policy reaction to real GDP. When the Japanese economy was faced with sluggish growth, the call lending rate was raised. Since the inflation rate was the main target of central bank monetary policy in the early 1980s. Figure 2 (4) shows monetary policy reaction to the lagged exchange rate. During the early 1980s, despite the yen's appreciation the call lending rate was raised in order to fight higher inflation. Figure 2 (5) is the fluctuation of the constant term. In our paper, it suggests the level of the call lending rate. In the early 1980s, the level of the call lending rate was high. On the other hand, it had been lowered during the bubble period and early 1990s. The response of the interest rate by the central bank shows no major reaction to any indicator since the late 1990s until recently.

In Fig. 3, the results of Eq. (33) are shown. Figure 3 (1) is the reaction of the inflation rate to the current nominal interest rate. A higher rate of inflation raised the current nominal interest rate in the 1980s and early 1990s. Figure 3 (2) is the response to the lagged interest rate of inflation. In the early 1980s, tight monetary policy led to a lower rate of inflation which is described as a negative coefficient of the lagged interest rate. Figure 3 (3) is the response to the lagged rate of inflation. During the asset bubble period and early 1990s negative coefficients were the case. This suggests that despite a positive expected rate of inflation, the actual rate of inflation was declining. Figure 3 (4) is the response of lagged real GDP to the rate of inflation. In the mid-1980s, relatively higher growth of the economy brought higher inflation. On the other hand, the influence of real GDP on inflation was quite small. Figure 3 (5) is the response of the lagged exchange rate to the rate of inflation. In the early 1980s, the rate of inflation continued to rise despite higher appreciation of the yen due to the second oil crisis. In the mid-1980s, high appreciation of the yen brought a lower rate of inflation which was one of the causes of the asset price bubble in Japan.

Figure 4 shows the aggregate demand function. Figure 4 (3) shows the response of the lagged interest rate on aggregate demand. In 1986, a lower interest rate pushed economic growth. However, in the 1990s, lower interest rates resulted in continuation of the sluggish economy which is denoted by a positive sign of the lagged interest rate on real GDP. Figure 4 (4) is the response of lagged inflation on aggregate demand. In 1983-85, the real economy was growing. However, in the 1990s, the inflation rate turned negative despite a positive low growth rate. The most recent period shows a strong positive sign since the inflation rate and economic growth have both turned positive. Figure 4 (5) is the response of lagged real GDP on aggregate demand. Figure 4 (6) is the response of a lagged exchange rate on aggregate demand. In 1986, despite the appreciation of the yen, real GDP was rising. In 1994, rapid appreciation of the yen brought slower growth. Figure 5 shows that during the bubble period inflation had been lowered and at the same time inflation was stable. Higher economic growth brought appreciation of the yen.

In Fig. 5, the results of Eq. (35) are shown. The coefficients of interest rates (namely i_t and i_{t-1}) show a positive sign for the entire period since appreciation of the yen lowers the interest rate. The coefficients of the inflation rate (namely π_t and π_{t-1}) show positive and negative signs for the entire period. Since the rate of inflation, the real interest rate is lowered. Thus, output increases and capital flows in from abroad, making yen appreciate. The coefficients of the inflation rate (namely and), and , show positive and negative signs for the entire period. Since output increases, stock prices are expected to rise. Thus, capital inflows increase and the yen appreciates.

4 Conclusions and Discussions

In this paper we give an empirical analysis of Japanese monetary policy using time-varying structural vector autoregressions (TVSVAR) based on the Monte Carlo particle filter and a self-organizing state space model. We estimated the time-varying reaction function of Japanese monetary policy, and our TVSVAR also included an aggregate supply function, an aggregate demand function, and nominal exchange rate determination function. Our TVSVAR is a dynamic full recursive structural VAR, similar to Primiceri (2005), Canova and Gambetti (2006), and many related papers. We would like to stress that Eq. (1) is a general formulation of time-varying-coefficient regression/autoregression modeling. While most previous studies are based on the Markov chain Monte Carlo method and the Kalman filter, we adopt a new TVSVAR estimation method, proposed by Yano (2008). The method is based on the Monte Carlo particle filter, proposed by Kitagawa (1996) and Gordon et al. (1993), and a self-organizing state space model, proposed by Kitagawa (1998), Yano (2007b), and Yano (2007a). In this paper we assume that the time evolution of coefficients is given by Markov chain processes. We call this assumption the Markov chain prior. The Markov chain prior is the generalization of the random walk prior. Thus, the main feature of our method is fewer restrictions than previous methods. Our method is applied for the estimation of a quarterly model of the Japanese economy (a nominal short-term interest rate, inflation rate, growth rate of real output, and change of the nominal effective exchange rate). We detect structural changes in most coefficients of TVSVAR.

The effectiveness of monetary policy using interest rates can be seen in aggregate supply from 1990. Interest rate policy toward aggregate demand is even worse in the sense that lower interest rate reduced output further. This paper concludes that Japan's sluggish economy was caused not only by aggregate supply factors but also by aggregate demand factors. The ineffectiveness of monetary policy from 1990 meant that the Japanese economy could not recover until recently.

For our future study, we would like to try the higher order of TVSVAR, estimating the time-varying coefficients of exogenous variables, and various types of TVSVAR. Moreover, we would like to try time-varying structural VAR with sign restrictions (TVSVAR-SR). TVSVAR-SR is a dynamic version of Uhlig (2005) [22].

Appendix A Data Source

We use quarterly macroeconomic data of the Japanese economy from 1980:Q1 to 2006:Q3.

- Uncollateralized overnight call rate, averaged over three months (Bank of Japan): uncollateralized overnight call rate, monthly average (July 1985-September 2006) and collateralized overnight call rate, monthly average (January 1980 - July 1985) are linked at July 1985.
  http://www.boj.or.jp/en/theme/research/stat/market/index.htm
- Seasonally-adjusted real/nominal GDP (Cabinet Office): quarterly estimates of GDP, chained, (1994:Q1-2006:Q3) and quarterly estimates of GDP, fixed-based, (1980:Q1-1994:Q1) are linked at 1994:Q1.
  http://www.esri.cao.go.jp/en/sna/menu.html
- Seasonally-adjusted GDP deflator (Cabinet Office): the deflator is calculated from seasonally-adjusted real/nominal GDP.
  http://www.esri.cao.go.jp/en/sna/menu.html
- Nominal effective exchange rate (Bank of Japan):
  http://www.boj.or.jp/en/theme/research/stat/market/forex/index.htm

[22] Braun and Shioji (2006) and Kamada and Sugo (2006) analyze the Japanese economy using sign-restricted VAR.

Appendix B Sensitivity Analysis

For sensitivity analysis, we estimate TVSVAR (1) with a different order of variables: a nominal interest rates, real growth rate, inflation rate, and nominal effective exchange rate. The results are shown in Fig. 11, 12, 13, and 14. Interestingly, Fig. 3 (2) and Fig. 13 (3) are nearly identical. Both figures indicate that BOJ's conduct of monetary policy by changing interest rates worked well to control the inflation rate. However, it has not worked to control the inflation rate since the 1990s. Moreover, Fig. 4 (3) and Fig. 12 (2) are nearly identical, too. It also shows monetary policy of BOJ worked well to control real GDP in the 1980s. However, it has not worked to control real GDP since the 1990s.

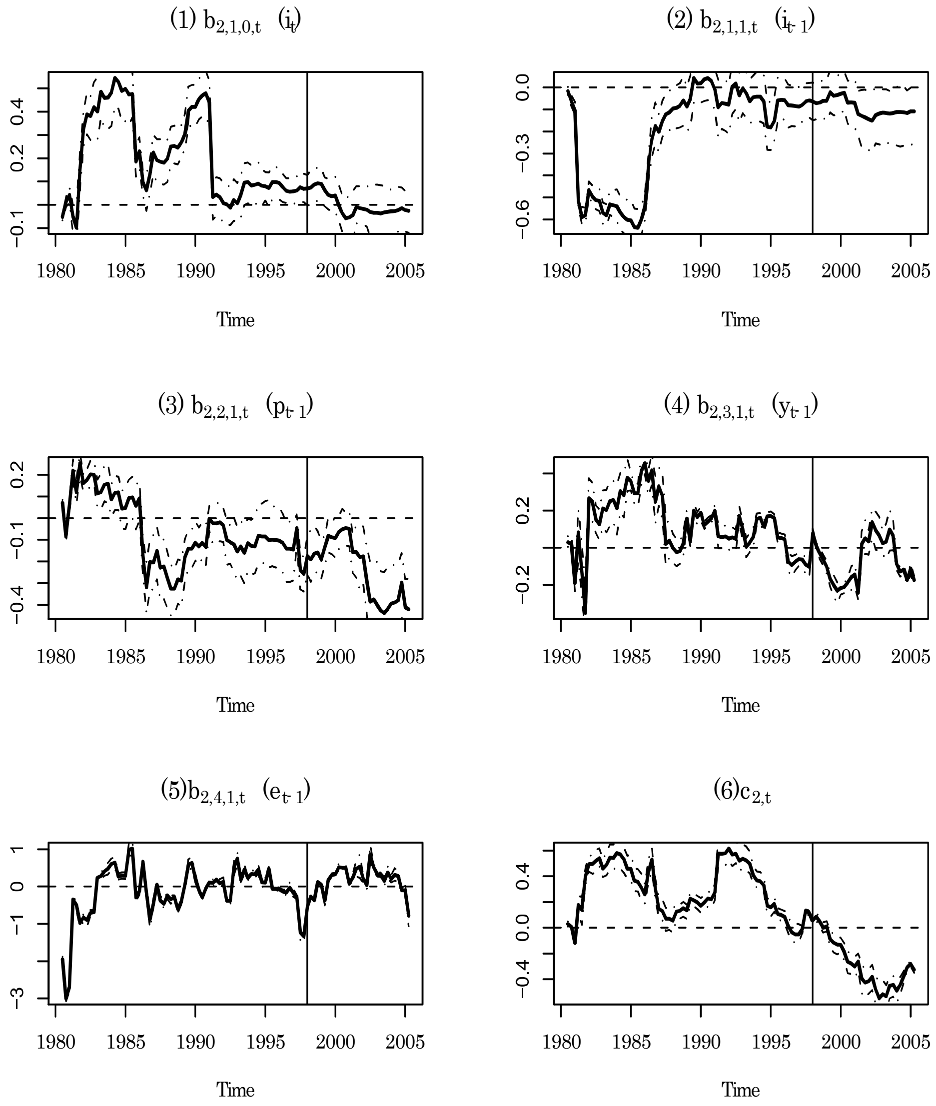

Figure 3: $\pi_t = b_{2;1;0;t} i_t + b_{2;1;1;t} i_{t-1} + b_{2;2;1;t}\pi_{t-1} + b_{2;3;1;t} y_{t-1} + b_{2;4;1;t} e_{t-1} + c_{2;t} + \epsilon_{2;t}$

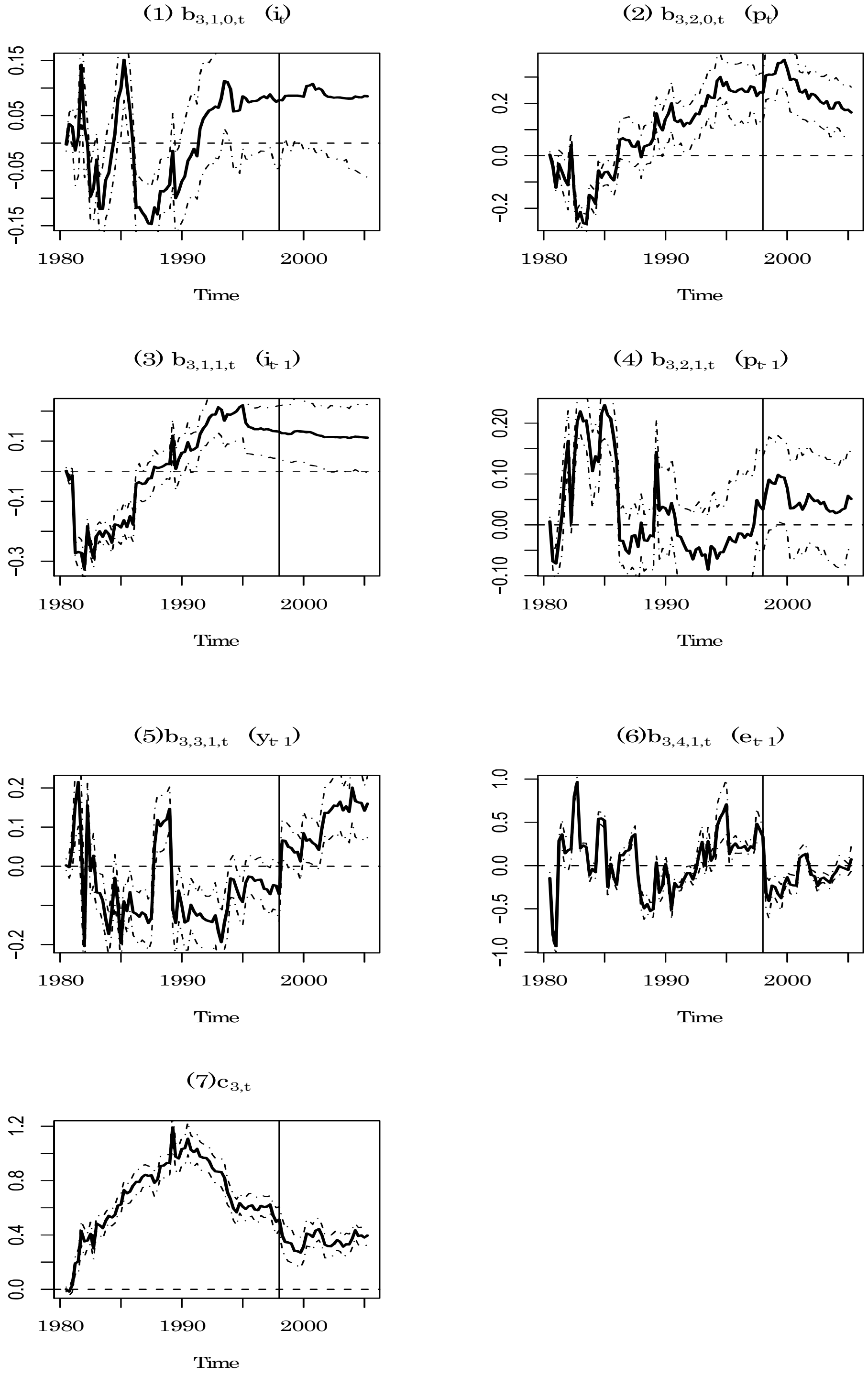

Figure 4: $y_t = b_{3;1;0;t} i_t + b_{3;2;0;t} \pi_t + b_{3;1;1;t} i_{t-1} + b_{3;2;1;t} \pi_{t-1} + b_{3;3;1;t} y_{t-1} + b_{3;4;1;t} e_{t-1} + c_{3;t} + \epsilon_{3;t}$

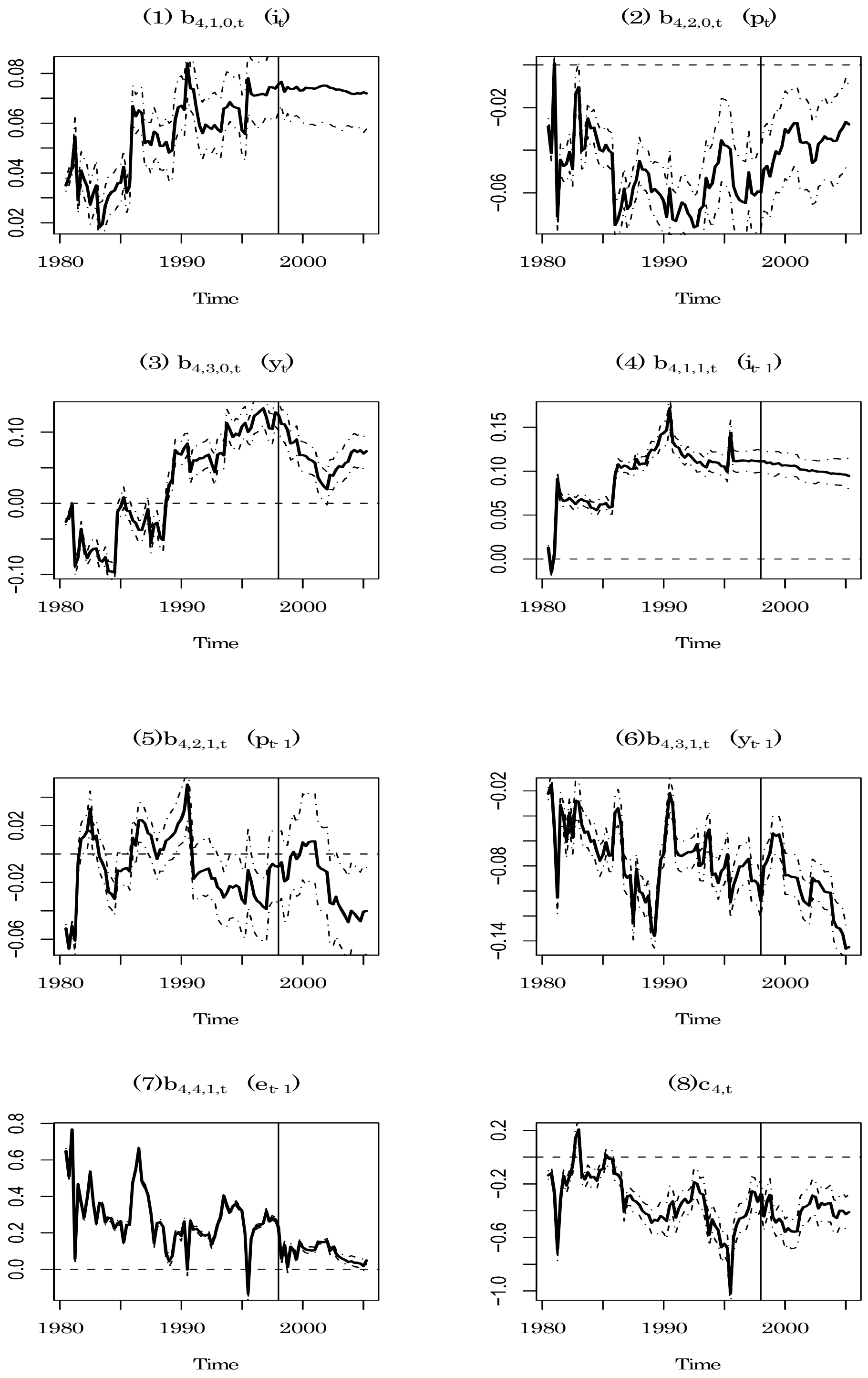

Figure 5: $e_t = b_{4;1;0;t} i_t + b_{4;2;0;t} \pi_t + b_{4;3;0;t} y_t + b_{4;1;1;t} i_{t-1} + b_{4;2;1;t} \pi_{t-1} + b_{4;3;1;t} y_{t-1} + b_{4;4;1;t} e_{t-1} + c_{4;t} + \epsilon_{4;t}$

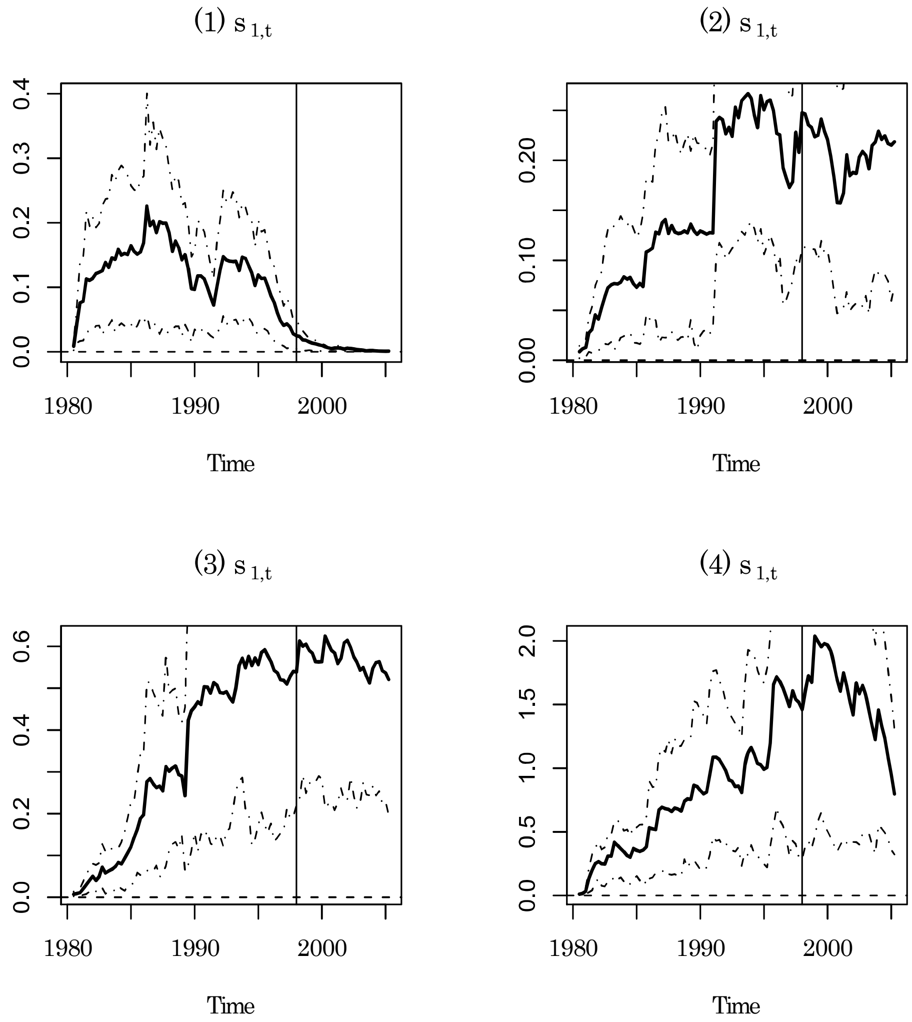

Figure 6: Stochastic Volatility (Full Recursive TVSVAR)

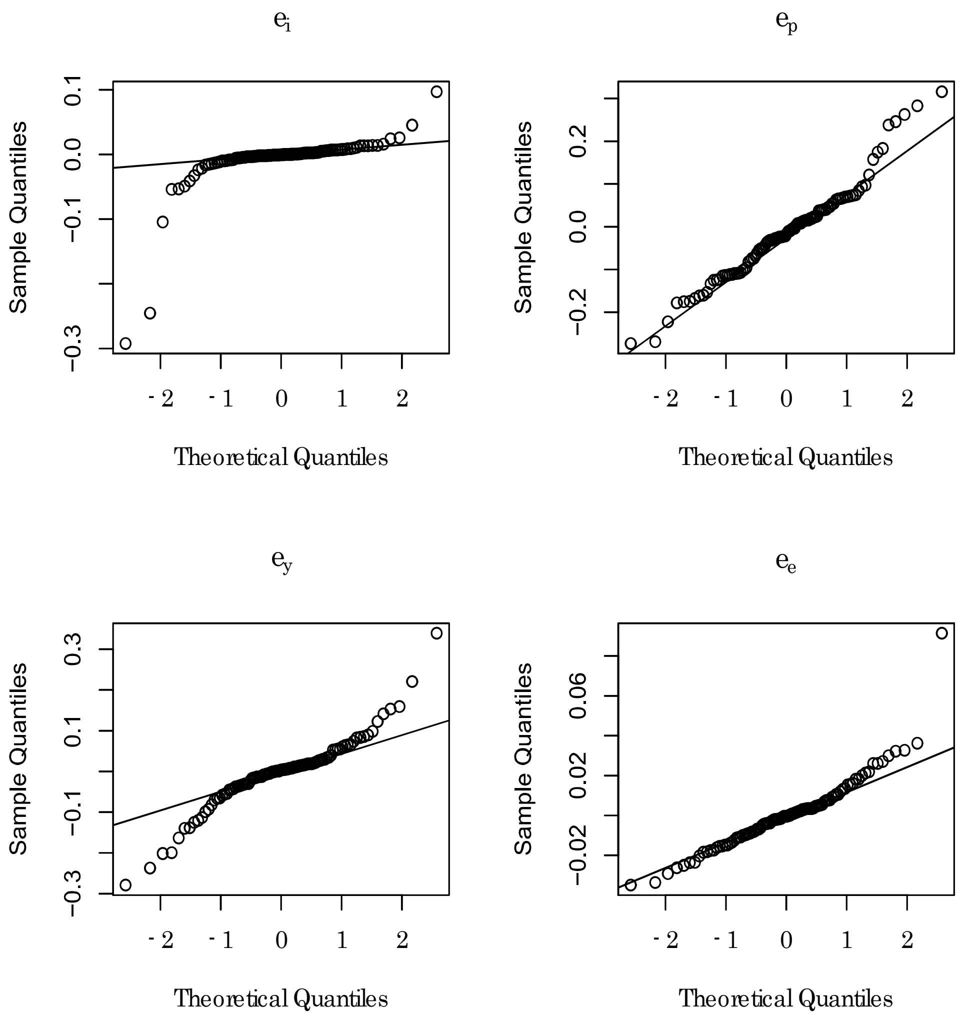

Figure 7: Q-Q Plot (Full Recursive TVSVAR)

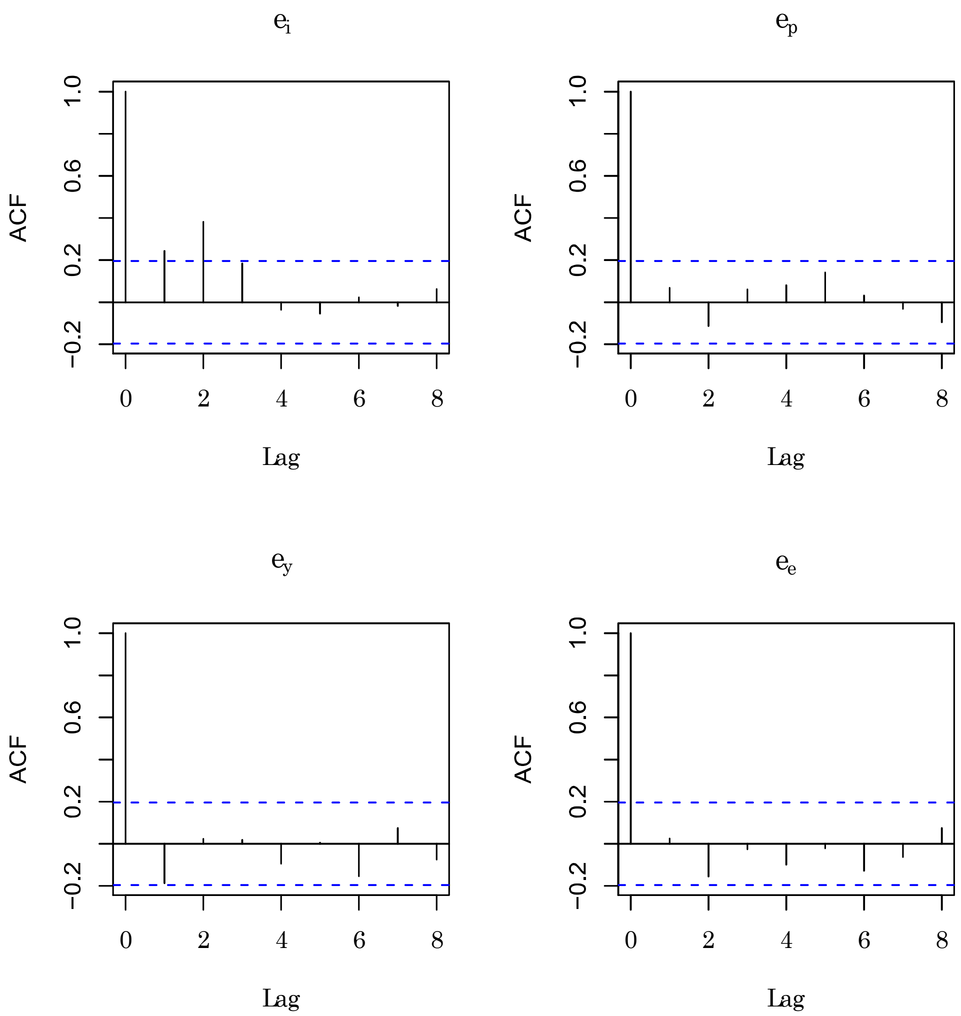

Figure 8: Autocorrelation with 95% Confidence Interval (Full Recursive TVSVAR)

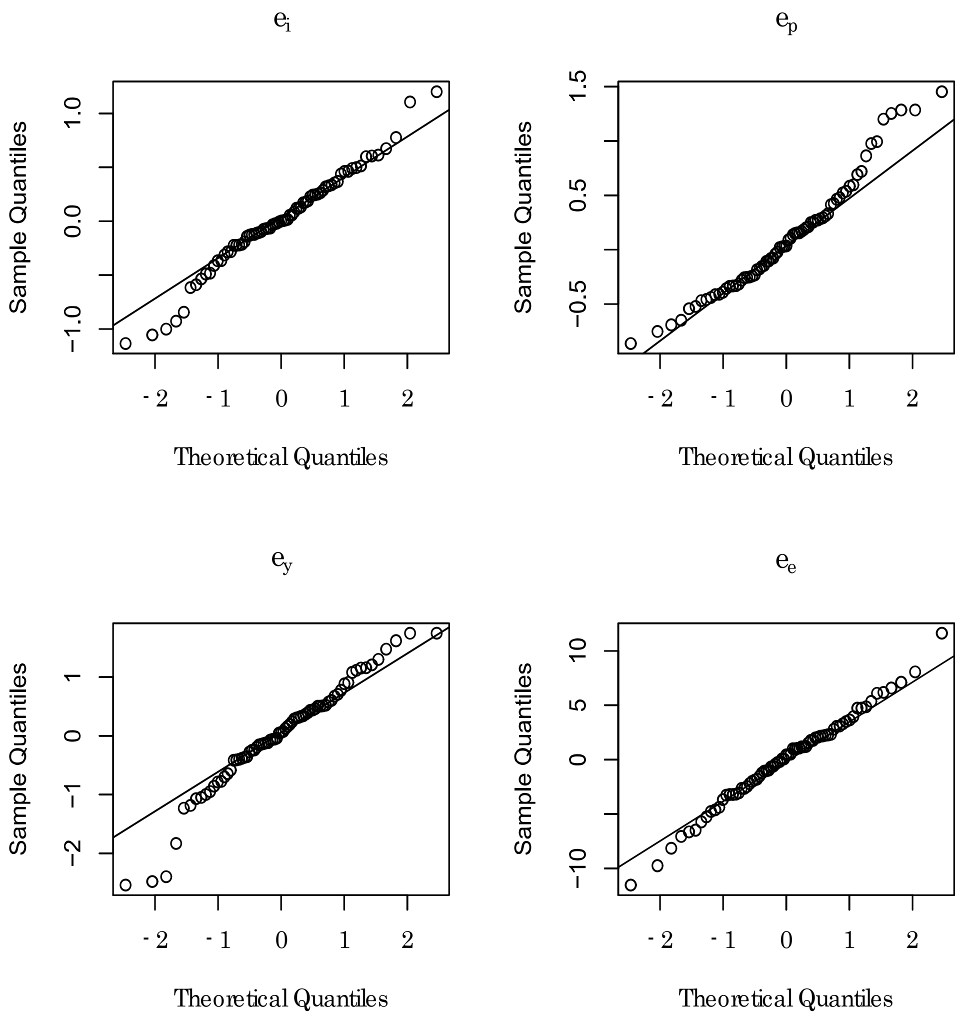

Figure 9: Q-Q Plot (Full Recursive SVAR)

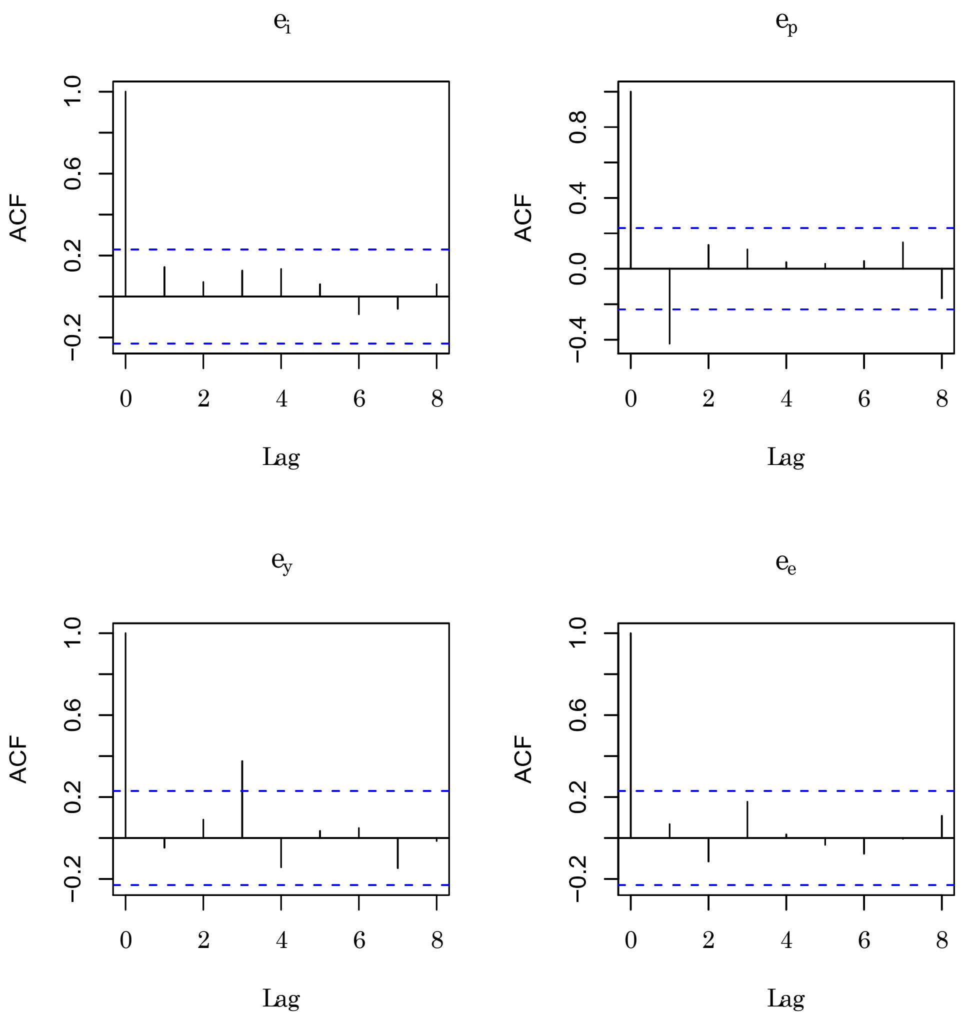

Figure 10: Autocorrelation with 95% Confidence Interval (Full Recursive SVAR)

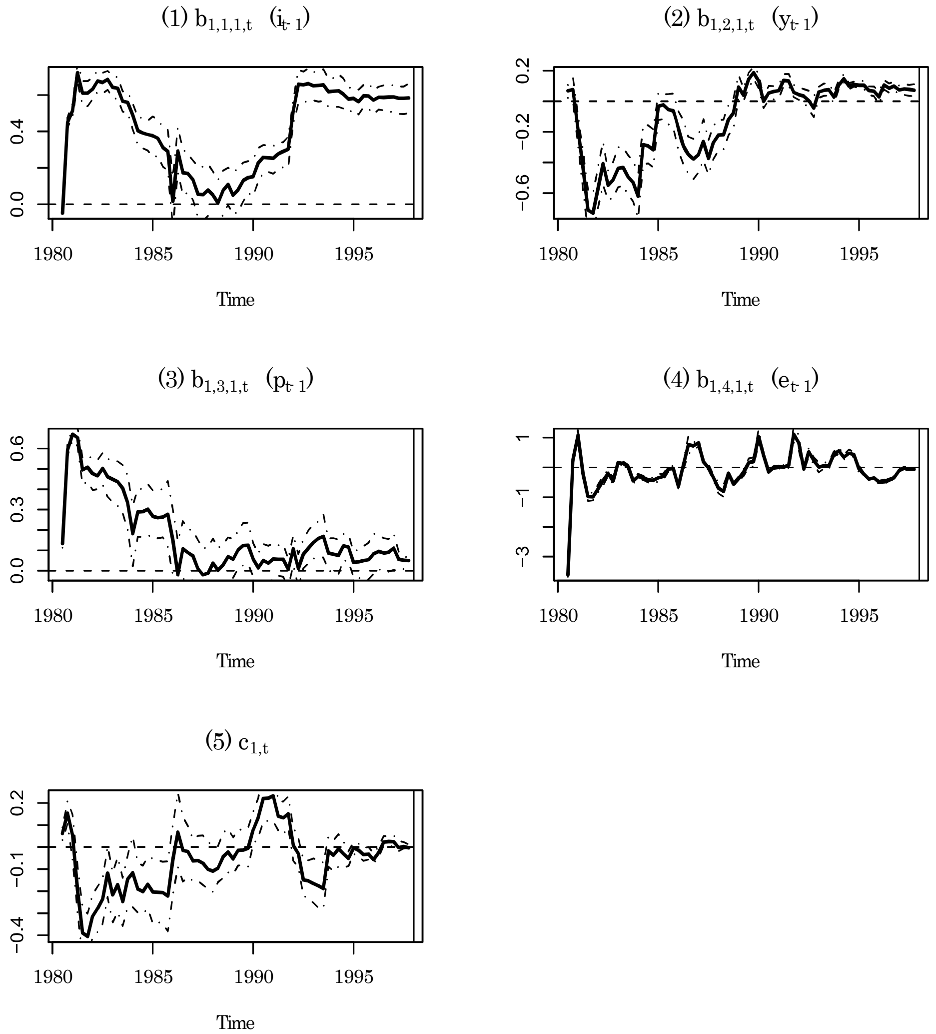

Figure 11: $i_t = b_{1;1;1;t}i_{t-1} + b_{1;2;1;t}y_{t-1} + b_{1;3;1;t}\pi_{t-1} + b_{1;4;1;t}e_{t-1} + c_{1;t} + \epsilon_{1;t}$

Appendix C Full Recursive TVSVAR (from 1980:Q1 to 1998:Q4)

We estimate TVSVAR based on quarterly data of the Japanese economy from 1980:Q1 to 1998:Q4 to avoid the zero-interest rate policy and quantitative easing policy periods. Fig. 15, 16, 17, and 18 show Eq. (32), (33), (34), and (35), respectively. In these figures, the solid line is an estimate of a time-varying coefficient and the dashed lines are 68% confidence intervals [23]. These figures are nearly identical to the figures in section 3. We conclude that it is very little to avoid the zero-interest rate policy and quantitative easing policy periods.

In Fig 19 and 20, we show a quantile-quantile plot and the autocorrelation of residuals of FR-TVSVR(1), respectively.

[23]Confidence interval is calculated using 100 times estimation of a time-varying coefficient.

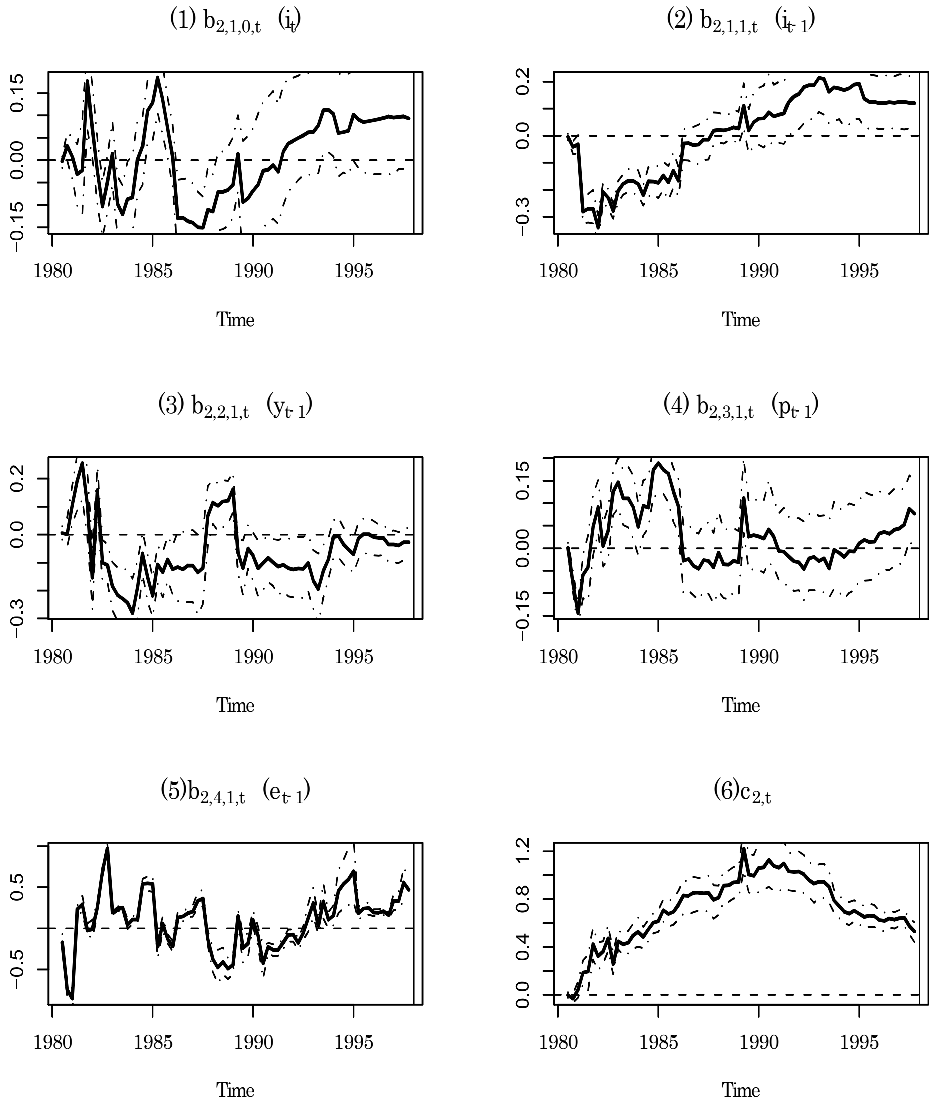

Figure 12: $y_t = b_{2;1;0;t}i_t + b_{2;1;1;t}i_{t-1} + b_{2;2;1;t}y_{t-1} + b_{2;3;1;t}\pi_{t-1} + b_{2;4;1;t}e_{t-1} + c_{2;t} + \epsilon_{2;t}$

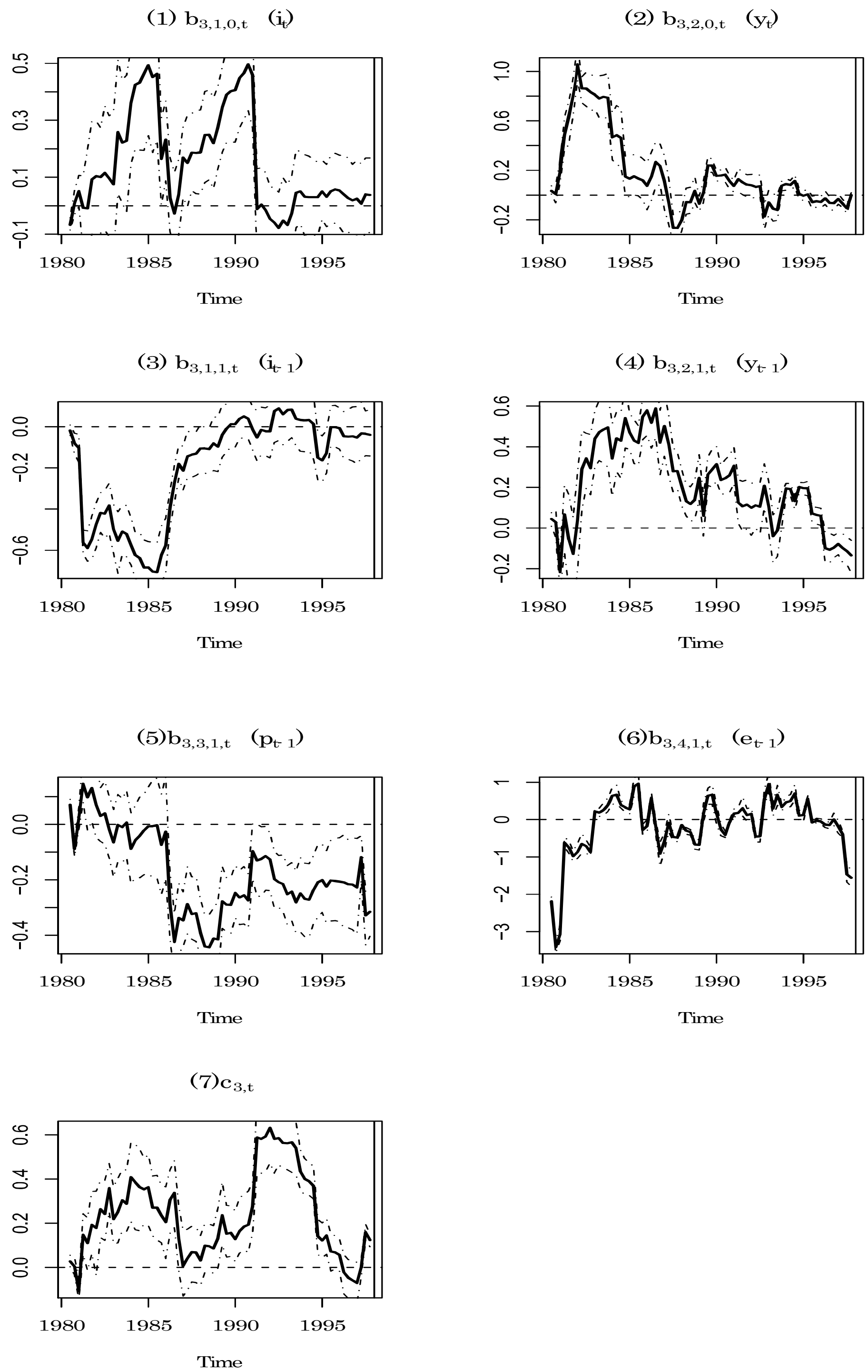

Figure 13: $\pi_t = b_{3;1;0;t} i_t + b_{3;2;0;t} y_t + b_{3;1;1;t} i_{t-1} + b_{3;2;1;t} y_{t-1} + b_{3;3;1;t} \pi_{t-1} + b_{3;4;1;t} e_{t-1} + c_{3;t} + \epsilon_{3;t}$

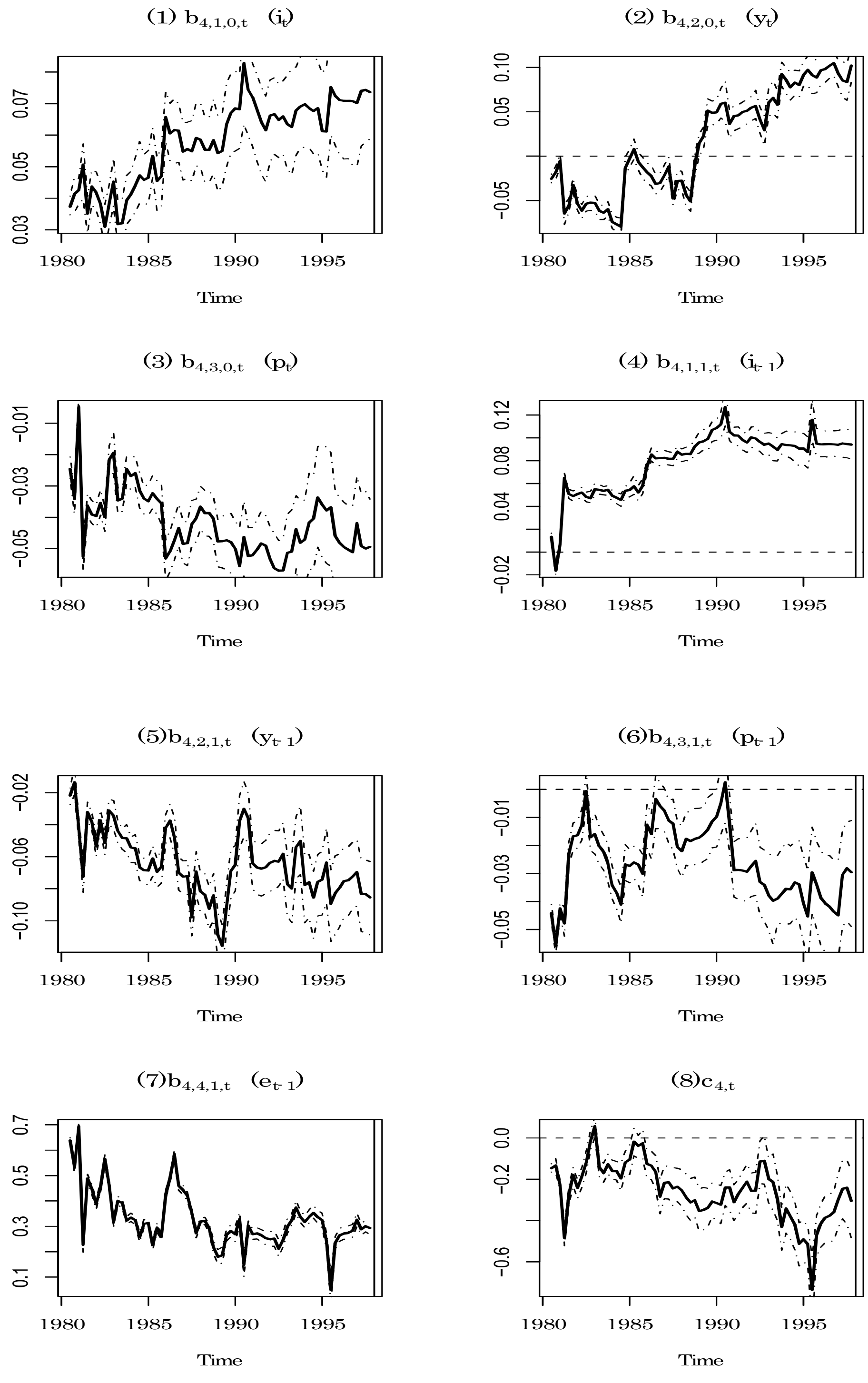

Figure 14: $e_t = b_{4;1;0;t} i_t + b_{4;2;0;t} y_t + b_{4;3;0;t} \pi_t + b_{4;1;1;t} i_{t-1} + b_{4;2;1;t} y_{t-1} + b_{4;3;1;t} \pi_{t-1} + b_{4;4;1;t} e_{t-1} + c_{4;t} + \epsilon_{4;t}$

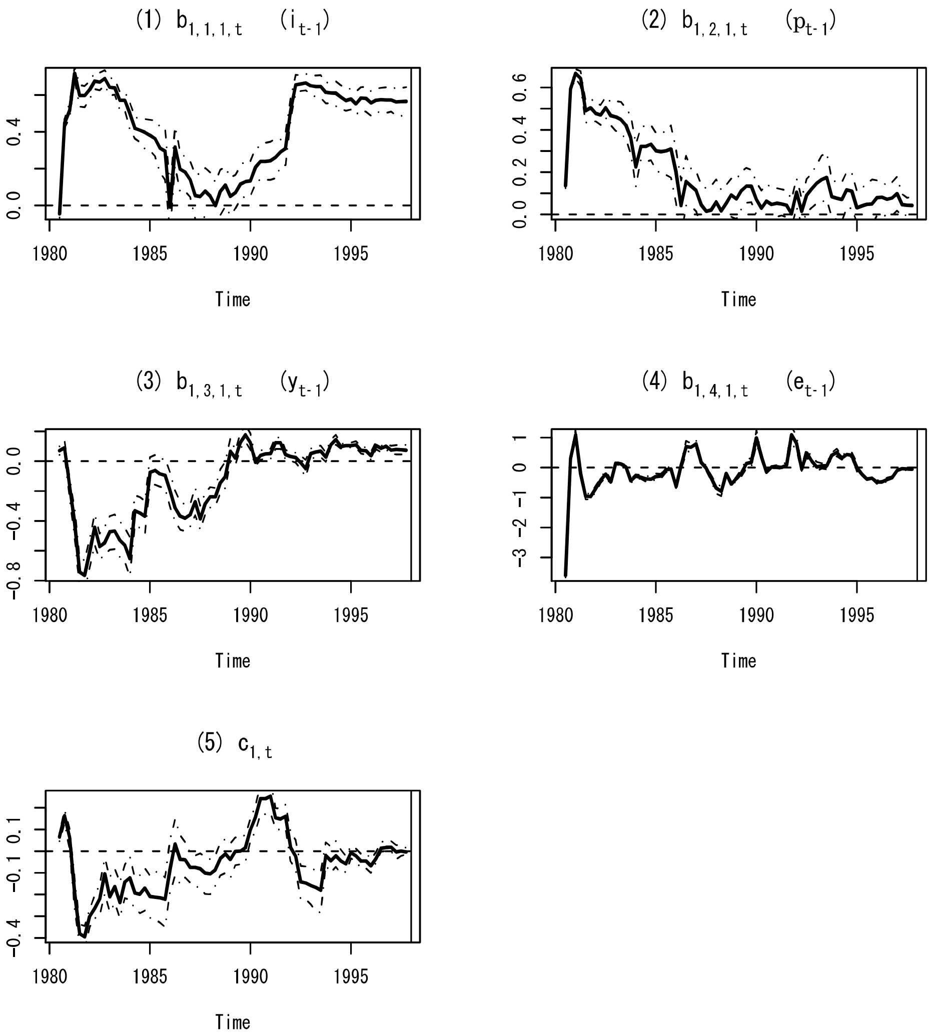

Figure 15: $i_t = b_{1;1;1;t} i_{t-1} + b_{1;2;1;t} \pi_{t-1} + b_{1;3;1;t} y_{t-1} + b_{1;4;1;t} e_{t-1} + c_{1;t} + \epsilon_{1;t}$

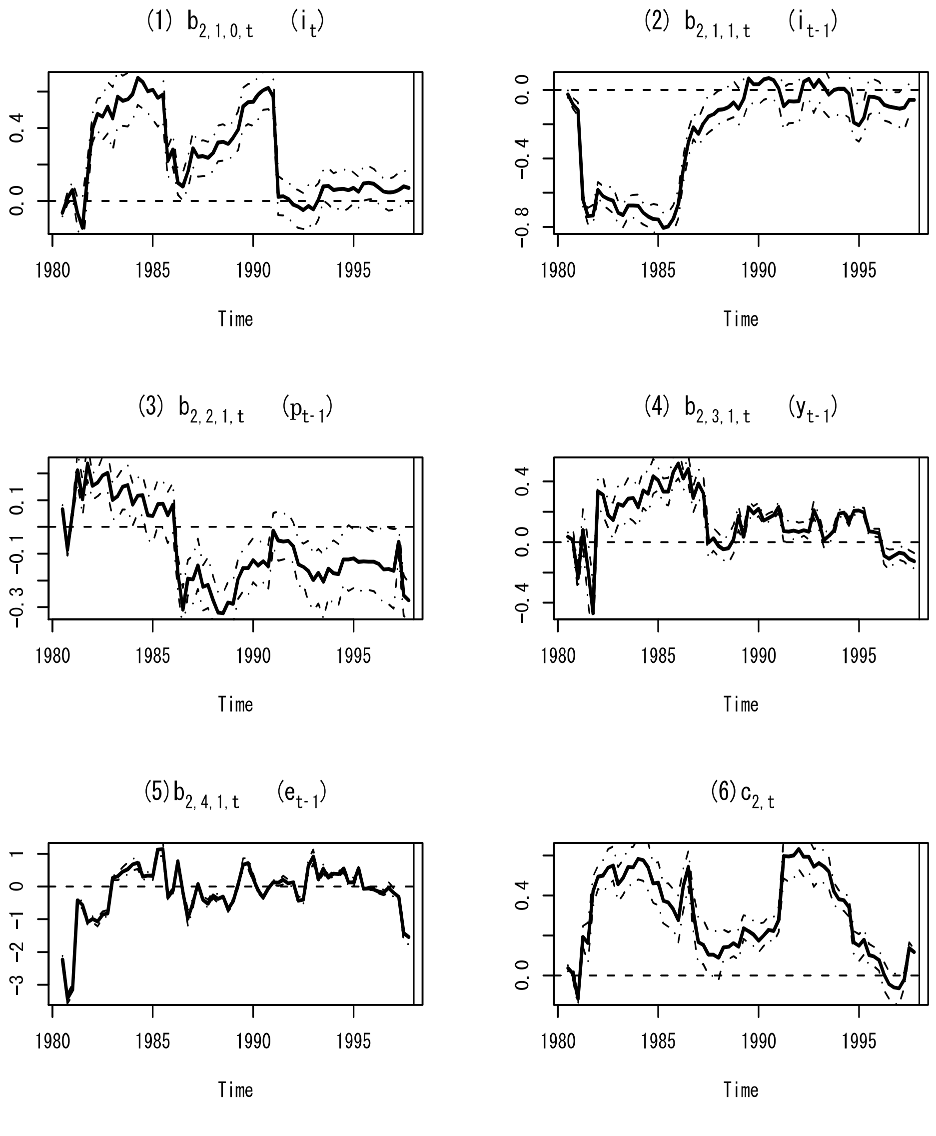

Figure 16: $\pi_t = b_{2;1;0;t} i_t + b_{2;1;1;t} i_{t-1} + b_{2;2;1;t} \pi_{t-1} + b_{2;3;1;t} y_{t-1} + b_{2;4;1;t} e_{t-1} + c_{2;t} + \epsilon_{2;t}$

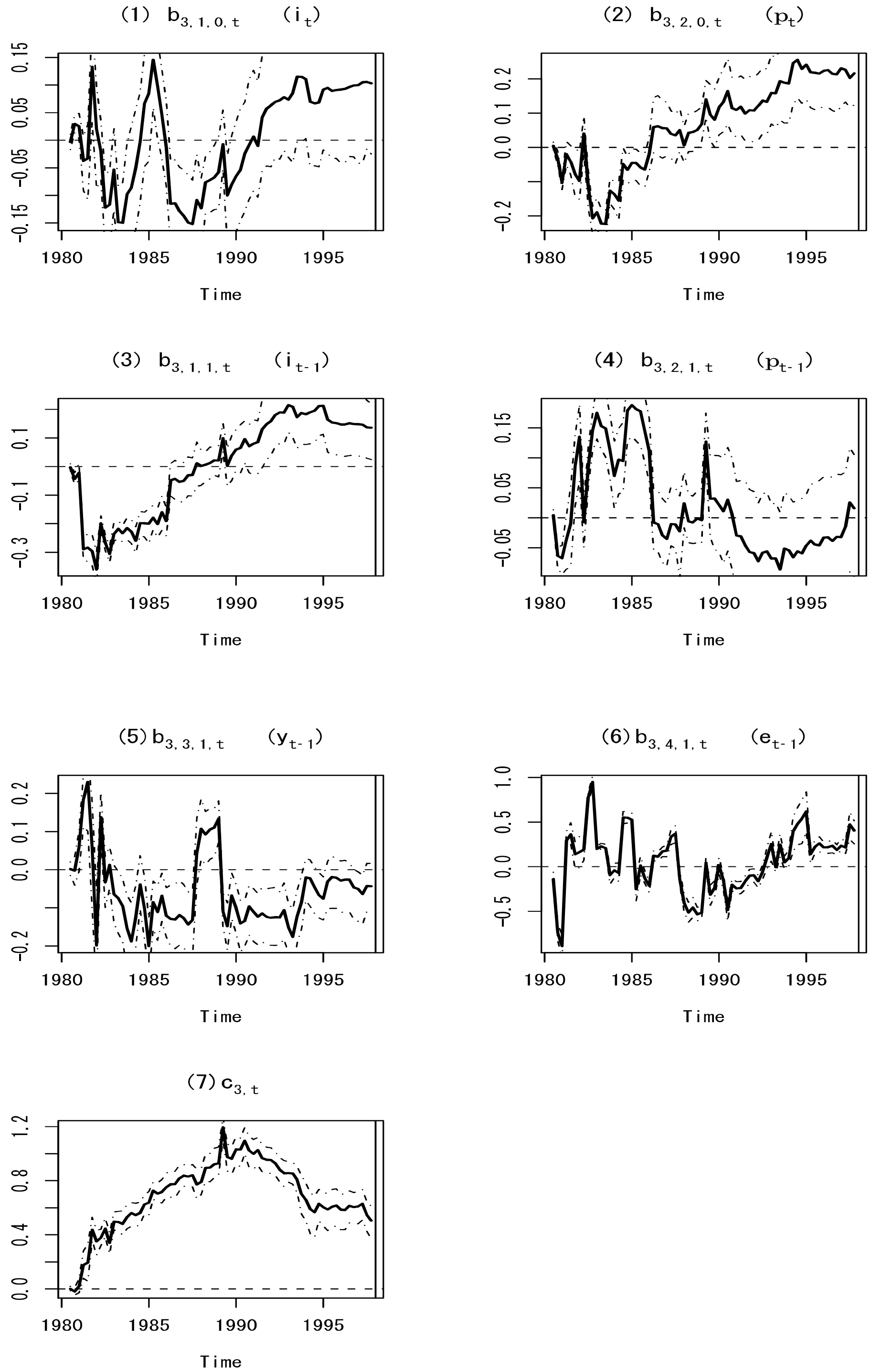

Figure 17: $y_t = b_{3;1;0;t} i_t + b_{3;2;0;t} \pi_t + b_{3;1;1;t} i_{t-1} + b_{3;2;1;t} \pi_{t-1} + b_{3;3;1;t} y_{t-1} + b_{3;4;1;t} e_{t-1} + c_{3;t} + \epsilon_{3;t}$

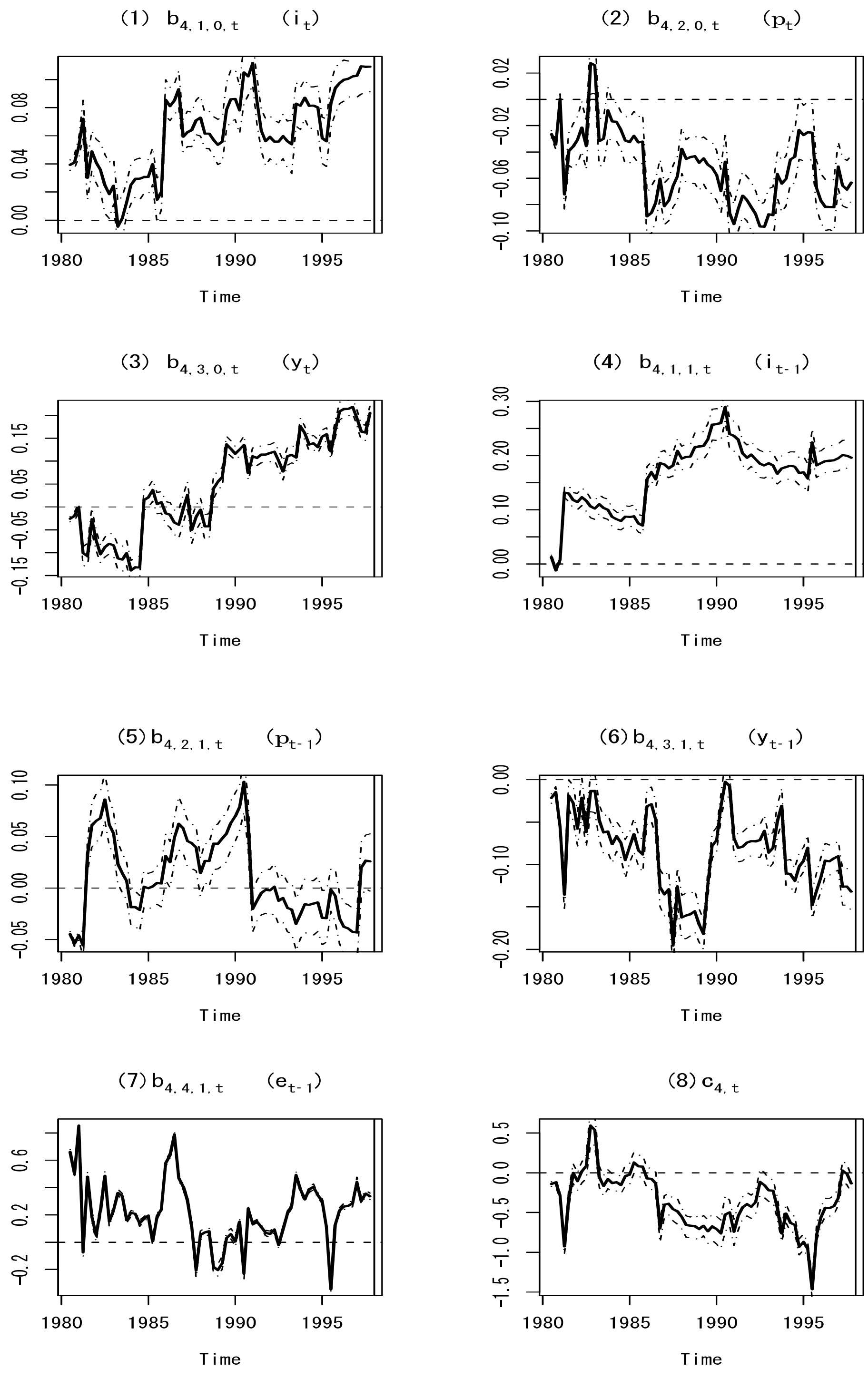

Figure 18: $e_t = b_{4;1;0;t}i_t + b_{4;2;0;t}\pi_t + b_{4;3;0;t}y_t + b_{4;1;1;t}i_{t-1} + b_{4;2;1;t}\pi_{t-1} + b_{4;3;1;t}y_{t-1} + b_{4;4;1;t}e_{t-1} + c_{4;t} + \epsilon_{4;t}$

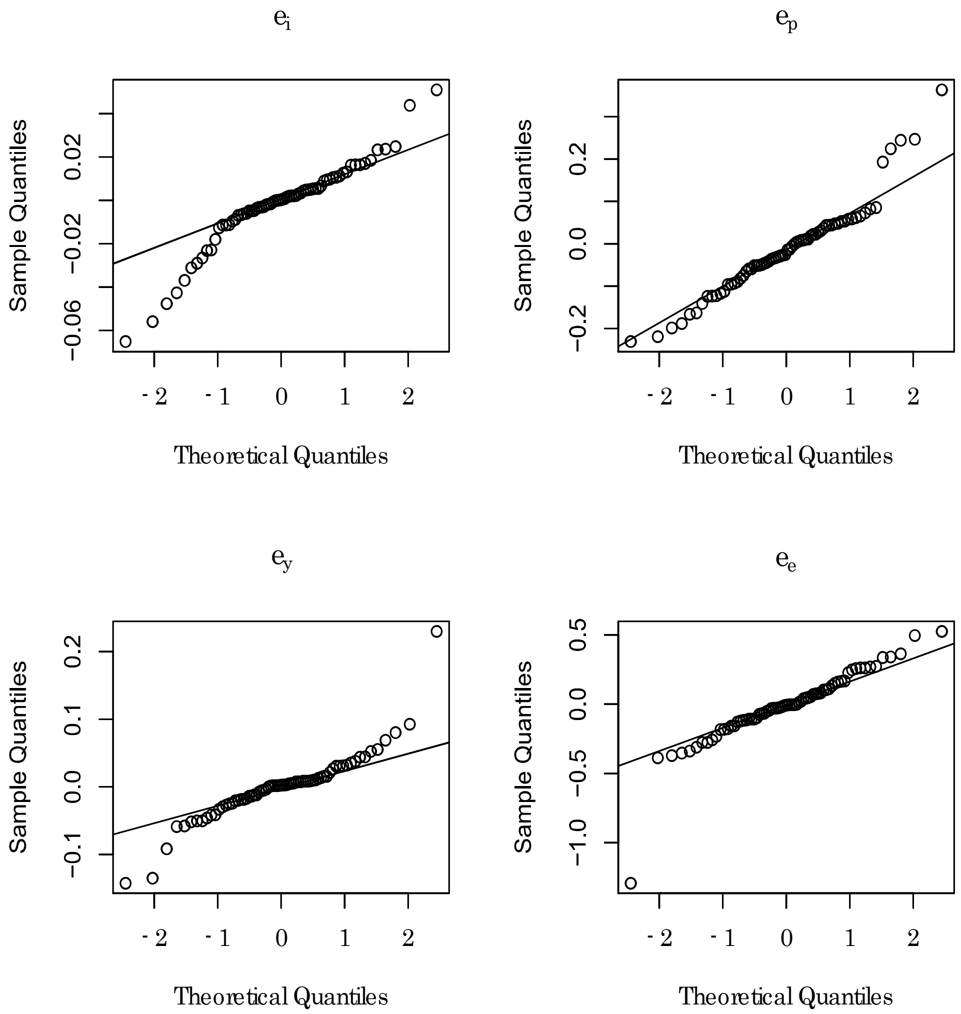

Figure 19: Q-Q Plot (Full Recursive TVSVAR)

We estimate the first order of SVAR (SVAR(1)) using quarterly data of the Japanese economy from 1980:Q1 to 1998:Q4 to avoid the zero-interest rate policy and quantitative easing policy periods. B_0 of SVAR(1) is

$$B_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0648 & 1 & 0 & 0 \\ -0.5243 & 0.0319 & 1 & 0 \\ -0.6978 & 0.1627 & -0.8602 & 1 \end{bmatrix}. \tag{C1}$$

The standard error of B_0 is

$$B_0^{SE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2515 & 1 & 0 & 0 \\ 0.2519 & 0.2241 & 1 & 0 \\ 0.2604 & 0.2242 & 0.1259 & 1 \end{bmatrix}. \tag{C2}$$

Eq. (C2) shows that most of the standard errors in B_0 are larger than the elements of B_0. It indicates the estimates of SVAR (1) are unreliable. In Table 2, we show the root mean square error of FR-TVSVAR(1) and SVAR(1).

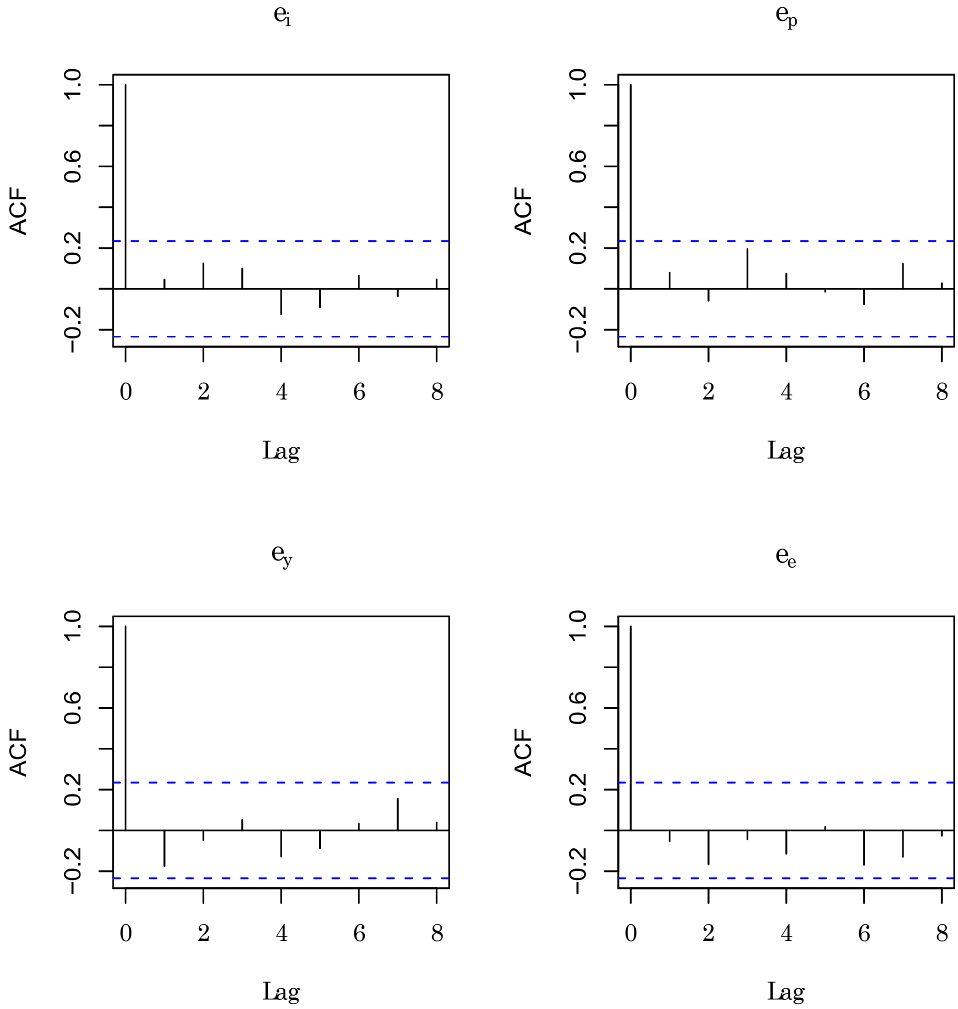

Figure 20: Autocorrelation with 95% Confidence Intervals (Full Recursive TVSVAR)

In Fig 21 and 22, we show a quantile-quantile plot and the autocorrelation of residuals of the first order of full recursive structural vector autoregressions, respectively.

Table 2: Root Mean Square Error

RMSE	Time-varying SVAR	Invariant-coefficient SVAR
ϵ_{i}	0.01950	0.44802
ϵ_{π}	0.10686	0.50403
ϵ_{y}	0.04861	0.89696
ϵ_{e}	0.24998	4.15722

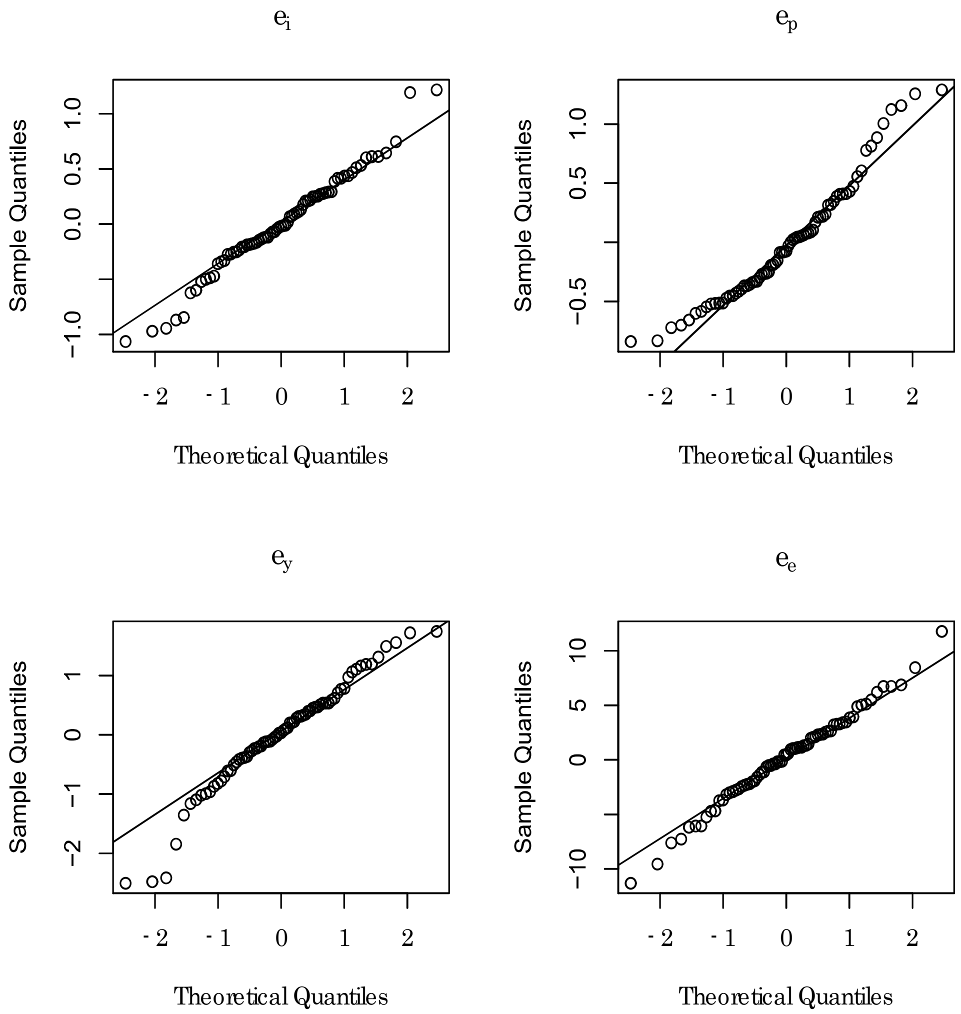

Figure 21: Q-Q Plot (Full Recursive SVAR)

Appendix D Non-Gaussian State Space Model

The time-varying coefficients $b_{i,j,l,t}$ and $d_{i,n,t}$ are estimated by using MCPF. The non-Gaussian state space representation is given by

$$
\begin{aligned}
x_t &= F x_{t-1} + G v_t, \\
y_{i,t} &= H_t x_t + \epsilon_{i,t}, \\
i &= 1, 2, \cdots, k,
\end{aligned} \tag{D3}
$$

where F, G, H_t are $(L \times L)$, $(L \times L)$, and $(1 \times L)$ matrices, respectively. x_t is an $(L \times 1)$ vector of coefficients, v_t is an L variate possibly non-Gaussian noise, $\epsilon_{i,t}$ is a possibly non-Gaussian noise, and $y_{i,t}$ an observation. The symbol L is $kp + n + i - 1$. Details of these vectors and matrices are explained in the following paragraphs. In our algorithm, matrices F, G are specified as follows.

$$
F = I_L, \quad G = I_L, \tag{D4}
$$

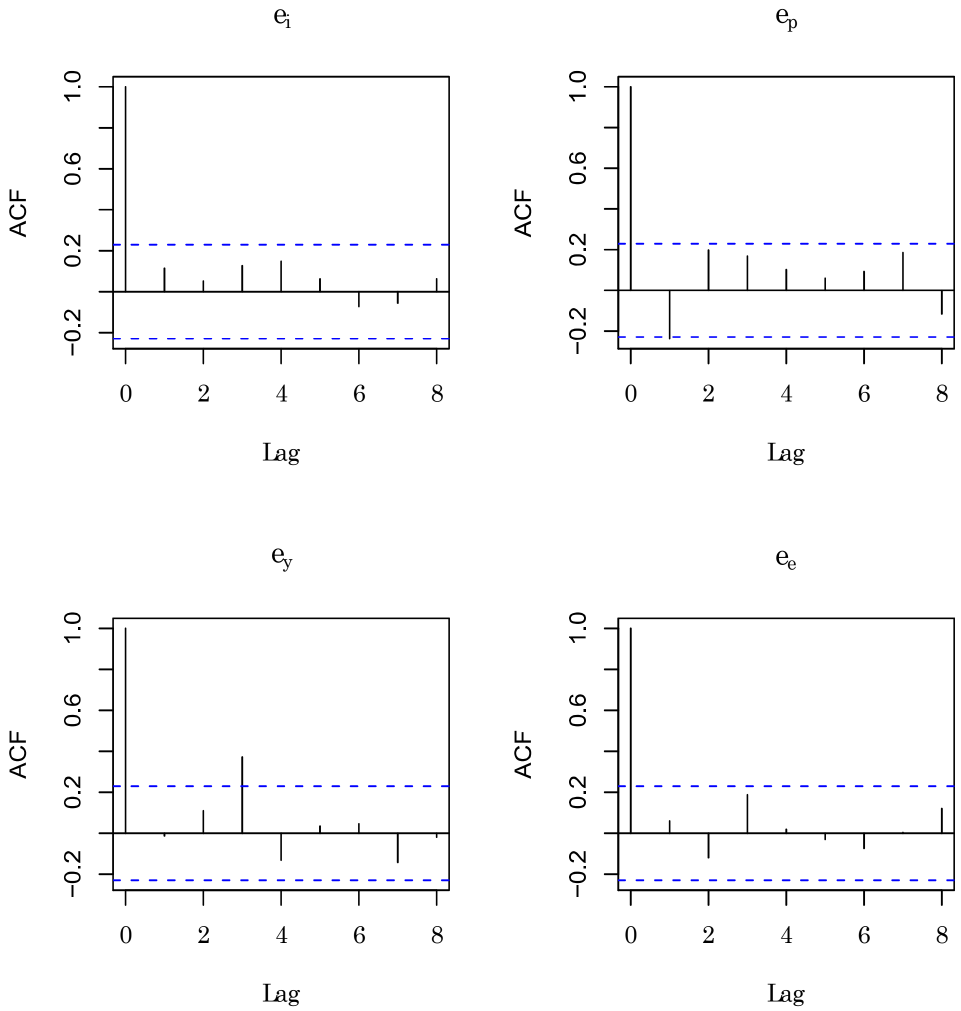

Figure 22: Autocorrelation with 95% Confidence Interval (Full Recursive SVAR)

where I_L is an L-dimensional identity matrix.

For the convenience of the expression, we use the following notations:

$$
\begin{aligned}
\hat{b}_{i;0;t} &= (b_{i;1;0;t}; \cdots ; b_{i;i-1;0;t});\\
\hat{b}_{i;t} &= (b_{i;1;1;t}; b_{i;2;1;t}; \cdots ; b_{i;k;1;t};\\
&\qquad b_{i;1;2;t}; \cdots ; b_{i;k;2;t}; \cdots ; b_{i;1;p;t}; \cdots ; b_{i;k;p;t});\\
\hat{d}_{i;t} &= (d_{i;1;t}; d_{i;2;t}; \cdots ; d_{i;n;t});\\
\hat{r}_{i;t} &= (y_{1;t}; y_{2;t}; \cdots ; y_{i-1;t});\\
\hat{h}_t &= (y_{1;t}; y_{2;t-1}; \cdots ; y_{k;t-1}; \cdots ;\\
&\qquad y_{1;t-p}; y_{2;t-p}; \cdots ; y_{k;t-p});\\
\hat{f}_t &= (u_{1;t-\kappa}; u_{2;t-\kappa}; \cdots ; u_{n;t-\kappa}):
\end{aligned}
\tag{D5}
$$

Vectors x_t and H_t are defined as follows. For the first component of y_t, $i = 1$,

$$\begin{aligned} x_t &= (\hat{b}_{1;t}; \hat{d}_{1;t})^T; \\ H_t &= (\hat{h}_t; \hat{f}_t): \end{aligned} \tag{D6}$$

For the ith component of $y(t)$, $1 < i \leq k$,

$$\begin{aligned} x_t &= (\hat{b}_{i;0;t}; \hat{b}_{i;t}; \hat{d}_{i;t})^T; \\ H_t &= (\hat{r}_{i;t}; \hat{h}_t; \hat{f}_t): \end{aligned} \tag{D7}$$

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