# Japanese Monetary Policy Reaction Function and Time-Varying Structural Vector Autoregressions: A Monte Carlo Particle Filtering Approach \*

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### Abstract

Recent years, the Japanese monetary policy is one of the hot topics on the Japan economy. This paper presents the empirical analysis on Japanese monetary policy based on Time-Varying Structural Vector AutoRegressions (TVSVAR). Our TVSVAR includes a monetary reaction function, an aggregate supply function, an aggregate demand function, and real exchange rate determination function. Our TVSVAR is a dynamic full recursive structural VAR, which is similar to Primiceri (2005), Canova and Gambetti (2006), and many related papers. The most of previous studies on TVSVAR are based on Markov Chain Monte Carlo method and the Kalman filter. We, however, adopt a new TVSVAR estimation method that is based on the Monte Carlo Particle filter and a self-organizing state space model, proposed by Kitagawa (1996), Gordon et al. (1993), Kitagawa (1998), Yano (2007b), and Yano (2007a). The method is proposed by Yano (2007c). Our methods are applied for the estimation of a quarterly model of the Japanese economy (a nominal short term interest rate, the rate of inflation, the growth rate of real output, and the real effective exchange rate). We would like to emphasize that our paper is the first one to analyze the Japanese economy using TVSVAR. It is often asked the causes of long term recession of the Japanese economy in 1990s whether it is caused by aggregate supply factor or aggregate demand factor. This paper concludes that both supply and demand factors contribute to the 10-years recession.

**Key words :** Monte Carlo particle filter, Self-organizing state space model, time-varying coefficient, structural vector autoregressions, Markov chain priors.

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# **1** Introduction

Recent years, Japanese monetary policy is one of the hot topics on the Japanese economy. This paper presents the empirical analysis on Japanese monetary policy using Time-Varying Structural Vector AutoRegressions (TVS-VAR). Our TVSVAR includes a monetary reaction function, an aggregate supply function, an aggregate demand function, and real exchange rate determination function. The changes of coefficients indicate the changes of the correlations of macroeconomic variables. Thus, we are able to analyze the changes of the Japanese economy. Our approach is related to Uhlig (1997), Cogley and Sargent (2001), Ciccarelli and Rebucci (2003), Cogley and Sargent (2005), Primiceri (2005), Sims and Zha (2006), Canova and Gambetti (2006), and many studies. The most of previous studies are based on Markov Chain Monte Carlo method and the Kalman filter<sup>4</sup>. In the studies, the random walk priors (the Minnesota priors), which are based on linear Gaussian state space modeling, are assumed on the time-evolutions of coefficients. The priors are proposed by Doan et al. (1984)<sup>5</sup>. Yano (2007c), however, proposes a new TVSVAR estimation method that is based on the Monte Carlo Particle filter and a self-organizing state space model, proposed by Kitagawa (1996), Gordon et al. (1993), Kitagawa (1998), Yano (2007b), and Yano (2007a). A novel feature of Yano (2007c) is that it assumes the time evolutions of coefficients are given by Markov chain processes. We call this assumption Markov chain priors on time-varying coefficients. Our priors are based on the nonlinear non-Gaussian state space modeling. The linear Gaussian cases of the Markov chain priors are equivalent to the random walk priors. Thus, our method is more flexible rather than previous methods. Our method is applied for the estimation of a quarterly model of the Japanese economy (a nominal short term interest rate, inflation rate, real growth rate, and the return of the real effective exchange rate). We detect structural changes in most coefficients of TVSVAR.

There exist previous studies on the Japanese monetary policy based on Bayesian statistical approach: Kimura et al. (2003), Fujiwara (2006), and Inoue and Okimoto (2007) <sup>6</sup>. Kimura et al. (2003) estimates time-varying reduced-form VAR models based on the Kalman filter. Fujiwara (2006) and Inoue and Okimoto (2007) analyze the regime changes of the Japanese economy in 1990s using Markov Switching VAR (MSVAR). The main advantages of our method to the previous studies are the we need less restrictions on the time-evolution of coefficients and less prior knowledge on structural changes. Kimura et al. (2003) assume random walk priors (the Minnesota priors) on the time-evolution of coefficients, which are based on linear Gaussian state space modeling. We, however, adopt *Markov chain priors*, which assume the the time-evolutions of coefficients follow Markov chain processes. Our assumption is less restricted rather than random walk priors. Fujiwara (2006) and Inoue and Okimoto (2007), use prior knowledge on the number of structural changes of the Japanese economy. In our method, the structural changes of coefficients of the economy are detected using the estimated time-varying coefficients of our model. Thus, we don't need prior knowledge on the structural changes of coefficients and the regime changes of the

<sup>&</sup>lt;sup>4</sup>Canova (2007) and Dejong and Dave (2007) are introductory textbooks on Bayesian statistical approach for macroeconomic analysis. Fernandez-Villaverde and Rubio-Ramirez (2005) and Fernandez-Villaverde and Rubio-Ramirez (2007) have shown that the Monte Carlo particle filter and Metropolis-Hastings algorithm can be successfully applied to estimate DSGE models.

<sup>&</sup>lt;sup>5</sup>The random walk priors are equivalent to first-order smoothness priors, proposed by Kitagawa (1983). TVSVAR based on the Kalman filter is adopted in Jiang and Kitagawa (1993) and Yano (2004) to estimate reduced-form time-varying coefficients vector autoregressions.

<sup>&</sup>lt;sup>6</sup>Miyao (2006) is a comprehensive survey on the Japanese macroeconomic and monetary policy based on structural VARs. Kasuya and Tanemura (2000) constructs Bayesian VAR optimized by the Posterior Information Criterion and estimates the performance of forecasting.

Japanese economy. Moreover, an advantage of our approach to the MSVAR approach is that it is not necessary for us to divide our date set into multi-pieces, even if several structural changes happen in the data. In the MSVAR approach, for example, if a structural change happens in your data set, you need to divide the data into two pieces to estimate each VAR for each piece. In general, the size of macroeconomic data is relatively small. Thus, this problem may cause poor estimation if several structural changes happen in your data set. In our approach, however, this problem doesn't happen. You are able to use your whole data set to estimate TVSVAR, even if several structural changes happen in it.

Major findings of this paper are summarized as follows. (i) Monetary policy by changing the interest rate worked well to control real GDP in 1980s. However, it did not work to control real GDP since 1990s. Furthermore lower interest rate brought lower economic growth. (ii) The rate of interest show almost no impact on rate of inflation after 1990 even though interest rate policy worked to control inflation in 1980s. (iii) Policy reaction of the interest rate to rate of inflation was strong in early 1980s. However the interest rate policy bounded by zero and the central bank of Japan could not set its interest rate into negative value. (iv) It is often asked the causes of long term recession of the Japanese economy in 1990s whether it is caused by aggregate supply factor (Hayashi and Prescott (2002), Hayashi (2003) and Miyao (2006)) or aggregate demand factor (such papers as Kuttner and Posen (2001) and Kuttner and Posen (2002)). This paper will show both supply and demand factors contributed to the 10-years recession. (v) From estimate of aggregate demand, there can be found spiral effects in Japanese economy and created downward spiral. (vi) From estimate of aggregate supply, the spiral effects are not observed in a sense that lower inflation did not cause much further decline in rate of inflation.

This paper is organized as follows. In section 2, we describe Time-Varying Structural Autoregressions and the outline of a new TVSVAR estimation method, proposed by Yano (2007c). In section 3, we show empirical analyses on the Japanese monetary policy and the Japanese economy. In section 4, we describe conclusions and discussions.

## 2 Time-Varying Structural Vector Autoregressions

In this section, we describe the outline of Yano (2007c). First, we describe Time-Varying Structural Vector Autoregressions and define state vectors to estimate it. Second, we explain the Monte Carlo particle filter and a self-organizing state space model to estimate a nonlinear non-Gaussian state space model.

#### 2.1 Time-Varying Structural Vector Autoregressions

Time-Varying Structural Vector Autoregressions (TVSVAR) for the time series  $Y_{1:T} = \{Y_1, Y_2, \dots, Y_T\}$  is defined as follows.

$$\boldsymbol{B}_{0,t}\boldsymbol{Y}_{t} = \sum_{p=1}^{F} \boldsymbol{B}_{p,t}\boldsymbol{Y}_{t-p} + \boldsymbol{D}_{t}\boldsymbol{u}_{t-\kappa} + \boldsymbol{c}_{t} + \boldsymbol{\epsilon}_{t}, \qquad (1)$$

where  $Y_t$  is a  $(k \times 1)$  vector of observations at time t,  $u_{t-\kappa}$  is an  $(n \times 1)$  vector of disturbances at time t,  $\kappa \ge 1$  is a constant,  $c_t$  is a  $(k \times 1)$  vector of time-varying intercepts at time t, and  $\epsilon_t = (\epsilon_{1,t}, \cdots, \epsilon_{k,t})^T \sim N(0, V)$  with

 $m{V}=diag(\sigma_1^2,\sigma_2^2,\cdots,\sigma_k^2)^{\ 7}$  . The matrices of time varying coefficients are

$$\boldsymbol{B}_{0,t} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ -b_{2,1,0,t} & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -b_{k,1,0,t} & \cdots & -b_{k,k-1,0,t} & 1 \end{bmatrix},$$
(2)

$$\boldsymbol{B}_{p,t} = \begin{bmatrix} b_{1,1,p,t} & \cdots & b_{1,k,p,t} \\ \vdots & \ddots & \vdots \\ b_{k,1,p,t} & \cdots & b_{k,k,p,t} \end{bmatrix},$$
(3)

and

$$\boldsymbol{D}_{t} = \begin{bmatrix} d_{1,1,t} & \cdots & d_{1,n,t} \\ \vdots & \ddots & \vdots \\ d_{k,1,t} & \cdots & d_{k,n,t} \end{bmatrix}.$$
(4)

Jiang and Kitagawa (1993) pointed out that Eq. (1) can be estimated by the each component of  $Y_t$  because V is a diagonal matrix. For example,  $Y_{1,t}$ , the first component of  $Y_t$  in Eq. (1), can be written by

$$Y_{1,t} = b_{1,1,1,t}Y_{1,t-1} + b_{1,2,1,t}Y_{2,t-1} \dots + b_{1,k,1,t}Y_{k,t-1} + \dots + b_{1,1,p,t}Y_{1,t-1} + b_{1,2,p,t}Y_{2,t-1} \dots + b_{1,k,p,t}Y_{k,t-1} + c_{1,t} + \epsilon_{1,t},$$
(5)

where  $c_{1,t}$  is the first component of  $c_t$ , and  $\epsilon_{1,t}$  is the first component of  $\epsilon_t$ . For another example,  $Y_{2,t}$ , the second component of  $Y_t$  in Eq. (1), can be written by

$$Y_{2,t} = b_{2,1,0,t}Y_{1,t} + b_{2,1,1,t}Y_{1,t-1} + b_{2,2,1,t}Y_{2,t-1} \dots + b_{2,k,1,t}Y_{k,t-1} + \dots + b_{2,1,p,t}Y_{1,t-1} + b_{2,2,p,t}Y_{2,t-1} \dots + b_{2,k,p,t}Y_{k,t-1} + c_{2,t} + \epsilon_{2,t},$$
(6)

where  $c_{2,t}$  is the second component of  $c_t$ , and  $\epsilon_{2,t}$  is the second component of  $\epsilon_t$ . For the first example, we define a state vector  $x_t$  of time varying coefficients as follows.

$$\boldsymbol{x}_{t} = \begin{bmatrix} b_{1,1,1,t}, b_{1,2,1,t}, \cdots, b_{1,k,1,t}, \cdots, b_{1,1,p,t}, b_{1,2,p,t}, \cdots b_{1,k,p,t}, c_{1,t} \end{bmatrix}^{T}.$$
(7)

For the second example, we define another state vector  $x_t$  of time varying coefficients as follows.

$$\boldsymbol{x}_{t} = \left[ b_{2,1,0,t}, b_{2,1,1,t}, b_{2,2,1,t}, \cdots, b_{2,k,1,t}, \cdots, b_{2,1,p,t}, b_{2,2,p,t}, \cdots, b_{2,k,p,t}, c_{2,t} \right]^{T}.$$
(8)

According to the discussion which is described above, the main problem of TVSVAR is how to estimate the state vector  $x_t$ . In the framework of sequential Bayesian filtering, the filtering distribution of  $x_t$ , which is based on the observations,  $Y_{1:t}$ , is given by

$$p(\boldsymbol{x}_t | \boldsymbol{Y}_{1:t}). \tag{9}$$

The smoothing distribution of  $x_t$ , which is based on the observations,  $Y_{1:T}$ , is given by

$$p(\boldsymbol{x}_t | \boldsymbol{Y}_{1:T}). \tag{10}$$

<sup>&</sup>lt;sup>7</sup>In this paper, a bold-faced symbol means a vector or a matrix.

Moreover, we assume that the time evolution of  $x_t$  is given by

$$p(\boldsymbol{x}_t | \boldsymbol{x}_{t-1}). \tag{11}$$

We refer to this assumption as *Markov Chain priors* on time-varying coefficients. Our priors are based on the nonlinear non-Gaussian state space modeling. The linear Gaussian cases of the Markov chain priors are equivalent to *random walk priors*, which are often adopted in previous studies. We would like to emphasize our Markov chain priors overcome the restriction of random walk priors. Our problem is how to estimate the state vector  $x_t$  using Eq. (9), (10), and (11). To solve the problem, we adopt the Monte Carlo particle filter. In the next subsection, we describe a method to estimate the state vector  $x_t$  using the filter.

# 2.2 Nonlinear Non-Gaussian State Space Modeling and A Self-Organizing State Space Model

To estimate a state vector  $x_t$ , we adopt the Monte Carlo Particle Filter (MCPF), proposed by Kitagawa (1996) and Gordon et al. (1993) and a self-organizing state space model, proposed by Kitagawa (1998). In this subsection, we describe a nonlinear non-Gaussian state space model and a self-organizing state space model (MCPF is described in the next subsection).

A nonlinear non-Gaussian state space model for the time series  $Y_t$ ,  $t = \{1, 2, \dots, T\}$  is defined as follows.

$$egin{aligned} & m{x}_t = f(m{x}_{t-1}) + m{v}_t, \ & m{Y}_t = h_t(m{x}_t) + m{\epsilon}_t, \end{aligned}$$

where  $x_t$  is an unknown  $n_x \times 1$  state vector,  $v_t$  is  $n_v \times 1$  system noise vector with a density function  $q(v|\cdot)^8$ ,  $\epsilon_t$  is  $n_{\epsilon} \times 1$  observation noise vector with a density function  $r(\epsilon|\cdot)$ . The function  $f : \mathbf{R}^{n_x} \times \mathbf{R}^{n_v} \to \mathbf{R}^{n_x}$  is a possibly nonlinear function and the function  $h_t : \mathbf{R}^{n_x} \times \mathbf{R}^{n_{\epsilon}} \to \mathbf{R}^{n_y}$  is a possibly nonlinear time-varying function. The first equation of (12) is called a system equation and the second equation of (12) is called an observation equation. A system equation depends on a possibly unknown  $n_s \times 1$  parameter vector,  $\boldsymbol{\xi}_s$ , and an observation equation depends on a possibly unknown  $n_o \times 1$  parameter vector,  $\boldsymbol{\xi}_o$ . This nonlinear non-Gaussian state space model specifies the two following conditional density functions.

$$p(\boldsymbol{x}_t | \boldsymbol{x}_{t-1}, \boldsymbol{\xi}_s),$$

$$p(\boldsymbol{Y}_t | \boldsymbol{x}_t, \boldsymbol{\xi}_o).$$
(13)

Note that  $p(x_t|x_{t-1}, \xi_s)$  is equivalent to Eq. (11). We define a parameter vector  $\theta$  as follows.

$$\boldsymbol{\theta} = \begin{bmatrix} \boldsymbol{\xi}_s \\ \boldsymbol{\xi}_o \end{bmatrix}. \tag{14}$$

We denote that  $\theta_j$  is the *j*th element of  $\theta$  and  $J(=n_s+n_o)$  is the number of elements of  $\theta$ . This type of state space model (12) contains a broad class of linear, nonlinear, Gaussian, or non-Gaussian time series models. In state space modeling, estimating the state space vector  $x_t$  is the most important problem. For the linear Gaussian state space model, the Kalman filter, which is proposed by Kalman (1960), is the most popular algorithm to estimate the state vector  $x_t$ . For nonlinear or non-Gaussian state space model, there are many algorithms. For example, the extended Kalman filter (Jazwinski (1970)) is the most popular algorithm and the other examples are the Gaussian-sum filter

<sup>&</sup>lt;sup>8</sup>The system noise vector is independent of past states and current states.

(Alspach and Sorenson (1972)), the dynamic generalized model (West et al. (1985)), and the non-Gaussian filter and smoother (Kitagawa (1987)). In recent year, MCPF for nonlinear non-Gaussian state space model is a popular algorithm because it is easily applicable to various time series models <sup>9</sup>.

In econometric analysis, generally, we don't know the parameter vector  $\theta$ . In the framework of TVSVAR, the unknown parameter vectors are  $\xi_o$  and  $\xi_s$ . In traditional parameter estimation, maximizing the log-likelihood function of  $\theta$  is often used. The log-likelihood of  $\theta$  in MCPF is proposed by Kitagawa (1996). However, MCPF is problematic to estimate the parameter vector  $\theta$  because the likelihood of the filter contains error from the Monte Carlo method. Thus, you cannot use nonlinear optimizing algorithm like Newton's method <sup>10</sup>. To solve the problem, Kitagawa (1998) proposes a self-organizing state space model. In Kitagawa (1998), an augmented state vector is defined as follows.

$$\boldsymbol{z}_t = \begin{bmatrix} \boldsymbol{x}_t \\ \boldsymbol{\theta} \end{bmatrix}. \tag{15}$$

An augmented system equation and an augmented measurement equation are defined as

$$z_t = F(z_{t-1}, v_t, \xi_s),$$

$$Y_t = H_t(z_t, \epsilon_t, \xi_o),$$
(16)

where

$$F(\boldsymbol{z}_{t-1}, \boldsymbol{v}_t, \boldsymbol{\xi}_s) = egin{bmatrix} f(\boldsymbol{x}_{t-1}) + \boldsymbol{v}_t, \ \boldsymbol{ heta} \ \boldsymbol{ heta} \end{bmatrix}$$

and

$$H_t(\boldsymbol{z}_t, \boldsymbol{\epsilon}_t, \boldsymbol{\xi}_o) = h_t(\boldsymbol{x}_t) + \boldsymbol{\epsilon}_t.$$

This nonlinear non-Gaussian state space model is called a self-organizing state space (SOSS) model. This selforganizing state space model specifies the two following conditional density functions.

$$p(\boldsymbol{z}_t | \boldsymbol{z}_{t-1}),$$

$$p(\boldsymbol{Y}_t | \boldsymbol{z}_t).$$
(17)

#### 2.3 The Monte Carlo Particle Filter

Most algorithms of sequential Bayesian filtering are based on Bayes' theorem (See Arulampalam et al. (2002)), which is

$$\mathbb{P}(\boldsymbol{z}_t | \boldsymbol{Y}_{1:t}) = \frac{\mathbb{P}(\boldsymbol{Y}_t | \boldsymbol{z}_t) \mathbb{P}(\boldsymbol{z}_t | \boldsymbol{Y}_{1:(t-1)})}{\mathbb{P}(\boldsymbol{Y}_t | \boldsymbol{Y}_{1:(t-1)})}, \ t \ge 1,$$
(18)

where  $\mathbb{P}(\boldsymbol{z}_t|\boldsymbol{Y}_{1:t-1})$  is the prior probability,  $\mathbb{P}(\boldsymbol{Y}_t|\boldsymbol{z}_t)$  is the likelihood,  $\mathbb{P}(\boldsymbol{z}_t|\boldsymbol{Y}_{1:t})$  is the posterior probability, and  $\mathbb{P}(\boldsymbol{Y}_t|\boldsymbol{Y}_{1:t-1})$  is the normalizing constant. We denote an initial probability  $\mathbb{P}(\boldsymbol{z}_0) = \mathbb{P}(\boldsymbol{z}_0|\emptyset)$ , where the empty set  $\emptyset$  indicates that we have no observations. In the state estimation problem, determining an initial probability  $\mathbb{P}(\boldsymbol{z}_0)$ , which is called filter initialization, is important because a proper initial probability improves a posterior probability. In TVSVAR, an initial probability is restricted in  $-1 < x_{i,0} < 1$ , where  $x_{i,0}$  is the *i*th element of  $\boldsymbol{x}_0$ .

In MCPF, the posterior density distribution at time t is approximated as

$$p(\mathbf{z}_t | \mathbf{Y}_{1:t}) = \frac{1}{\sum_{m=1}^{M} w_t^m} \sum_{m=1}^{M} w_t^m \delta(\mathbf{z}_t - \mathbf{z}_t^m),$$
(19)

<sup>&</sup>lt;sup>9</sup>Many applications are shown in Doucet et al. (2001).

<sup>&</sup>lt;sup>10</sup>See Yano (2007b).

where  $w_t^m$  is the weight of a particle  $z_t^m$ , M is the number of particles, and  $\delta$  is the Dirac's delta function <sup>11</sup>. The definition of  $w_t^m$  is described below. In the standard algorithm of MCPF, particles are resampled with sampling probabilities proportional to the weights  $w_t^m$  at every time t. It is necessary to prevent increasing the variance of weights after few iterations of Eq. (18) <sup>12</sup>. After resampling, the weights are reset to  $w_t^m = 1/M$ . Therefore, Eq. (19) is rewritten as

$$p(\boldsymbol{z}_t | \boldsymbol{Y}_{1:t}) = \frac{1}{M} \sum_{m=1}^{M} \delta(\boldsymbol{z}_t - \hat{\boldsymbol{z}}_t^m)$$
(20)

where  $\hat{z}_t^m$  are particles after resampling. Using Eq. (20), the predictor  $p(z_t|Y_{1:(t-1)})$  can be approximated by

$$p(\mathbf{z}_{t}|\mathbf{Y}_{1:(t-1)}) = \int p(\mathbf{z}_{t}|\mathbf{z}_{t-1})p(\mathbf{z}_{t-1}|\mathbf{Y}_{1:(t-1)})d\mathbf{z}_{t-1}$$

$$= \frac{1}{M} \sum_{m=1}^{M} \int p(\mathbf{z}_{t}|\mathbf{z}_{t-1})\delta(\mathbf{z}_{t-1} - \hat{\mathbf{z}}_{t-1}^{m})d\mathbf{z}_{t-1}$$

$$= \frac{1}{M} \sum_{m=1}^{M} p(\mathbf{z}_{t}|\hat{\mathbf{z}}_{t-1}^{m})$$

$$\simeq \frac{1}{M} \sum_{m=1}^{M} \delta(\mathbf{z}_{t} - \mathbf{z}_{t}^{m}).$$
(21)

Note that  $\boldsymbol{z}_t^m$  are obtained from

$$\boldsymbol{z}_t^m \sim p(\boldsymbol{z}_t | \hat{\boldsymbol{z}}_{t-1}^m). \tag{22}$$

Substituting Eq. (21) to Eq. (18), we obtain the following equation.

$$p(\boldsymbol{z}_{t}|\boldsymbol{Y}_{1:t}) \propto p(\boldsymbol{Y}_{t}|\boldsymbol{z}_{t})p(\boldsymbol{z}_{t}|\boldsymbol{Y}_{1:(t-1)})$$

$$\propto \frac{1}{M}p(\boldsymbol{Y}_{t}|\boldsymbol{z}_{t})\sum_{m=1}^{M}\delta(\boldsymbol{z}_{t}-\boldsymbol{z}_{t}^{m})$$

$$= \frac{1}{M}\sum_{m=1}^{M}p(\boldsymbol{Y}_{t}|\boldsymbol{z}_{t}^{m})\delta(\boldsymbol{z}_{t}-\boldsymbol{z}_{t}^{m}).$$
(23)

Comparing Eq. (19) and Eq. (23) indicates that weights  $w_t^m$  are obtain by

$$w_t^m \propto p(\boldsymbol{Y}_t | \boldsymbol{z}_t^m). \tag{24}$$

Therefore, a weight  $w_t^m$  is defined as

$$w_t^m \propto p(\boldsymbol{Y}_t | \boldsymbol{z}_t^m) = r(\psi_t(\boldsymbol{Y}_t, \boldsymbol{z}_t^m)) \Big| \frac{\partial \psi}{\partial y} \Big|, m = \{1, \cdots, M\},$$
(25)

where  $\psi_t$  is the inverse function of the function  $h_t^{13}$ . In our TVSVAR estimation method, the augmented state vector is estimated using MCPF. Thus, states and parameters are estimated simultaneously without maximizing the log-likelihood of Eq. (16) because the parameter vector  $\boldsymbol{\theta}$  in Eq. (16) is approximated by particles and it is

$$\delta(x) = 0$$
, if  $x \neq 0$   
 $\int_{\infty}^{\infty} \delta(x) dx = 1$ .

<sup>12</sup>See Doucet et al. (2000).

<sup>&</sup>lt;sup>11</sup>The Dirac delta function is defined as

<sup>&</sup>lt;sup>13</sup>See Kitagawa (1996).

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Algorithm: Time-Varying Structural Vector Autoregressions Estimation
	ext{SOSS}[\left\{m{z}_{t-1}^m
ight\}_{m=1}^M,m{y}_t]
{
  FOR m=1,...M
    Predict: \boldsymbol{z}_{t}^{m} \sim p(\boldsymbol{z}_{t} | \boldsymbol{z}_{t-1}^{m}, \boldsymbol{v}_{t}^{m})
     Weight: w_t^m is obtained by Eq. (25)
  ENDFOR
  Sum of Weights: sw = \sum_{m=1}^{M} w_t^i
  Log-Likelihood: llk = log(sw/M)
  FOR m=1,...,M
    Normalize: \hat{w}_t^m = \frac{w_t^m}{s^m}
  ENDFOR
  Resampling: [\{\hat{z}_t^m, \hat{w}_t^m\}_{m=1}^M] =resample[\{z_t^m, \hat{w}_t^m\}_{m=1}^M]
  RETURN[\{\hat{\boldsymbol{z}}_{t}^{m}, \hat{w}_{t}^{m}\}_{m=1}^{M}, llk]
}
SOSS.MAIN[\{x_0^m\}_{m=1}^M, \{y\}_{t=1}^T, P]
{
  \boldsymbol{\theta}_0 \sim uniform(\boldsymbol{P}-r, \boldsymbol{P}+r)
  \{z_0^m\}_{m=1}^M = (\{\pmb{x}_0^m\}_{m=1}^M, \{\theta_0^m\}_{m=1}^M)
  FOR t=1.....T
    soss = SOSS[\{\hat{\boldsymbol{z}}_{t-1}^m\}_{m=1}^M, \boldsymbol{y}_t]
    \{\hat{z}_{t}^{m}, \hat{w}_{t}^{m}\}_{m=1}^{M} = (\{\hat{z}_{t}^{m}, \hat{w}_{t}^{m}\}_{m=1}^{M} in \text{ soss})
  ENDFOR
  \texttt{RETURN}[\left\{\left\{ \boldsymbol{\hat{z}}_{t}^{m}, \ \boldsymbol{\hat{w}}_{t}^{m} \right\}_{m=1}^{M} \right\}_{t=1}^{T}, Y]
}
```

estimated as the state vector in Eq. (15)  $^{14}$ . The algorithm of our TVSVAR estimation method is summarized as follows  $^{15}$ .

On a self-organizing state space model, however, Hürseler and Künsch (2001) points out a problem: determination of initial distributions of parameters for a self-organizing state space model. The estimated parameters of a self-organizing state space model comprise a subset of the initial distributions of parameters. We must know the posterior distributions of parameters to estimate parameters adequately. However, the posterior distributions of the parameters are generally unknown. Parameter estimation fails if we do not know appropriate their initial distributions. Yano (2007b) proposes a method to seek initial distributions of parameters for a self-organizing state space model using the simplex Nelder-Mead algorithm to solve the problem. To seek initial distributions of parameters, we adopt the algorithm, which is proposed by Yano (2007b). Moreover, we adopt the smoothing algorithm and filter initialization method, which is proposed by Yano (2007a).

<sup>&</sup>lt;sup>14</sup>The justification of an SOSS model is described in Kitagawa (1998).

<sup>&</sup>lt;sup>15</sup>The details of MCPF and SOSS is described in Yano (2007b).

#### 2.3.1 Functional Forms

In this paper, we use linear non-Gaussian state space models to estimate time-varying coefficients and parameters. A linear non-Gaussian state is given by

$$egin{aligned} & m{x}_t = m{x}_{t-1} + m{v}_t, \ & (26) \ & Y_{i,t} = m{H}_t m{x}_t + \epsilon_{i,t}, \end{aligned}$$

where  $Y_{i,t}$  is an observation,  $v_t \sim q(v_t | \boldsymbol{\xi}_t)$ ,  $\epsilon_{i,t} \sim r_i(\epsilon_{i,t} | \boldsymbol{\xi}_{i,o})$ ,  $\epsilon_{i,t}$  is the *i*th component of  $\boldsymbol{\epsilon}_t$ , and  $\boldsymbol{\xi}_{i,o}$  is the *i*th component of  $\boldsymbol{\xi}_o$ . The details of  $\boldsymbol{x}_t$  and  $\boldsymbol{H}_t$  are described in Appendix C. In our Markov chain priors,  $q(v_t | \boldsymbol{\xi}_t)$  and  $r_i(\epsilon_{i,t} | \boldsymbol{\xi}_{i,o})$  are possibly non-Gaussian distributions. We would like to emphasize that our priors make the estimation of TVSVAR flexible rather than random walk priors. In this paper, the innovation term  $q(\boldsymbol{v}_t | \boldsymbol{\xi}_t)$  is specified by normal distributions or *t*-distributions, and  $r_i(\epsilon_{i,t} | \boldsymbol{\xi}_{i,o})$  is specified by normal distributions. In general, the components,  $\{\xi_{1,s}, \xi_{2,s}, \dots, \xi_{L,s}\}$ , of  $\boldsymbol{\xi}_s$  are different (L is defined in appendix C). In this paper, however, to reduce computational complexity, we assume as follows.

(A1) 
$$\xi_{1,s} = \xi_{2,s} = \dots = \xi_{L,s} = |\xi_s|$$
 (27)

$$A2) \quad \xi_s = \sigma_s \tag{28}$$

In this paper, the time evolutions of coefficients are given by

$$x_{i,t} = x_{i,t-1} + |\xi_s| \times t(df),$$
(29)

where df is the degree of freedom of Student's *t*-distribution. The innovation term of  $Y_i$  is given by the normal distributions  $(\epsilon_{i,t} \sim N(0, \sigma_s^2))^{16}$ .

(

### **3** Empirical Analyses

Our methods are applied for the estimation of a quarterly model of the Japanese economy. In the model, four variables are included : a short-term interest rate (the uncollateralized overnight call rate), the rate of inflation (the growth rate of seasonal adjusted GDP deflator), the growth rate of output (the growth rate of seasonal adjusted real GDP), and the return of the real effective exchange rate <sup>17</sup>. We use data from 1980:I up to 2006:III. Rate hikes are given by the first difference of the mean of the monthly average of the uncollateralized overnight call rate <sup>18</sup>. The growth rates of GDP deflator and real output are given by

$$x_t = \left[\log X_t - \log X_{t-1}\right] \times 100.$$
(31)

The growth rate of the real effective exchange rate is given by

$$e_t = -\left[\log E_t - \log E_{t-1}\right] \times 100,\tag{32}$$

where  $E_t$  is the real effective exchange rate. Note that  $e_t$  becomes smaller when Yen is appreciated.

In Fig. 1, the four variables are shown.

#### [Figure 1 about here.]

(B2) 
$$\sigma_{i,m,t} = |\sigma_{i,m,t-1} + \eta_t|,$$
 (30)

where  $\eta_t \sim N(0, \xi_{i,o}^2)$ .

<sup>&</sup>lt;sup>16</sup>Primiceri (2005) proposes TVSVAR with Stochastic Volatility. An elements of Time-Varying Variance Covariance Matrix is given by

<sup>&</sup>lt;sup>17</sup>The details of data are described in Appendix A.

<sup>&</sup>lt;sup>18</sup>See Miyao (2000), Miyao (2002), and Miyao (2006).

#### 3.1 Full Recursive TVSVAR

We estimate the first order of full recursive TVSVAR (FR-TVSVAR(1)) as a benchmark model. FR-TVSVAR(1) is given by

$$i_t = b_{1,1,1,t}i_{t-1} + b_{1,2,1,t}\pi_{t-1} + b_{1,3,1,t}y_{t-1} + b_{1,4,1,t}e_{t-1} + c_{1,t} + \epsilon_{i,t},$$
(33)

$$\pi_t = b_{2,1,0,t}i_t + b_{2,1,1,t}i_{t-1} + b_{2,2,1,t}\pi_{t-1} + b_{2,3,1,t}y_{t-1} + b_{2,4,1,t}e_{t-1} + c_{2,t} + \epsilon_{\pi,t},$$
(34)

$$y_t = b_{3,1,0,t}i_t + b_{3,2,0,t}\pi_t + b_{3,1,1,t}i_{t-1} + b_{3,2,1,t}\pi_{t-1} + b_{3,3,1,t}y_{t-1} + b_{3,4,1,t}e_{t-1} + c_{3,t} + \epsilon_{y,t},$$
(35)

$$e_{t} = b_{4,1,0,t}i_{t} + b_{4,2,0,t}\pi_{t} + b_{4,3,0,t}y_{t} + b_{4,1,1,t}i_{t-1} + b_{4,2,1,t}\pi_{t-1} + b_{4,3,1,t}y_{t-1} + b_{4,4,1,t}e_{t-1} + c_{4,t} + \epsilon_{e,t}.$$
(36)

where  $i_t$  is the first difference of the short-term interest rate,  $\pi_t$  is the rate of inflation,  $y_t$  is the growth rate of real output, and  $e_t$  is the return of the real effective exchange rate <sup>19</sup>. Following Miyao (2000), Miyao (2002), and Miyao (2006), the variables of FR-TVSVAR(1) are ordered from exogenous variables to endogenous variables <sup>20</sup>.

First, we estimate TVSVAR based on quarterly data of the Japanese economy from 1980:Q1 to 1998:Q4 to avoid the periods of zero-interest rate policy and quantitative easing policy. Fig. 2, 3, 4, and 5 show Eq. (33), (34), (35), and (36), respectively. In these figures, the solid line is a estimate of a time-varying coefficient and the dash-lines are 68% confidence interval <sup>21</sup>.

# [Figure 2 about here.] [Figure 3 about here.] [Figure 4 about here.] [Figure 5 about here.]

In Fig. 6, 7, 8, and 9, we show Impulse Response Functions (1980:Q4, 1985:Q4, 1989:Q4, 1997:Q4), respectively.

[Figure 6 about here.][Figure 7 about here.][Figure 8 about here.][Figure 9 about here.]

$$b_{x,y,z,t} = sd_{exp}/sd_{obs}$$

<sup>&</sup>lt;sup>19</sup>We set the number of particles, M, to 10000. Moreover, we set the degrees of freedom, df, in Eq. (29) to 10, 20, 30. In the all cases, we get same results. In our paper, we show results that we set df to 10. The other parameters of simulation are same in Yano (2007b) and Yano (2007a). All time-varying coefficients are standardized as follows:

where  $sd_{exp}$  is the standard deviation of an explaining variable and  $sd_{exp}$  is the standard deviation of an observation (this standardization method may not be best).

<sup>&</sup>lt;sup>20</sup>The results of block recursive TVSVAR, which is a dynamic version of SVAR, proposed by Christiano et al. (1999), are shown.

<sup>&</sup>lt;sup>21</sup>Confidence interval is calculated using 100 times estimation of a time-varying coefficient.

In Fig 10 and 11, we show Quantile-Quantile plot and autocorrelation of residuals of FR-TVSVR(1), respectively.

[Figure 10 about here.]

[Figure 11 about here.]

Second, we estimate TVSVAR based on quarterly data of the Japanese econmy from 1980:Q1 to 2006:Q3. Fig. 12, 13, 14, and 15 show Eq. (33), (34), (35), and (36), respectively.

[Figure 12 about here.][Figure 13 about here.][Figure 14 about here.][Figure 15 about here.]

In Fig 16 and 17, we show Quantile-Quantile plot and autocorrelation of residuals of FR-TVSVR(1), respectively.

[Figure 16 about here.]

[Figure 17 about here.]

We compare TVSVAR with (invariant coefficient) Structural VAR (SVAR) using residual analysis. SVAR(P) is given by

$$\boldsymbol{B}_{0}\boldsymbol{Y}_{t} = \boldsymbol{B}_{1}\boldsymbol{Y}_{t-1} + \boldsymbol{B}_{2}\boldsymbol{Y}_{t-2} + \dots + \boldsymbol{B}_{P}\boldsymbol{Y}_{t-P} + \boldsymbol{\epsilon}_{t}$$

First, we estimate the first order of SVAR (SVAR(1)) using quarterly data of the Japanese economy from 1980:Q1 to 1998:Q4 to avoid the periods of zero-interest rate policy and quantitative easing policy.  $B_0$  of SVAR(1) is

$$\boldsymbol{B}_{0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0648 & 1 & 0 & 0 \\ -0.5243 & 0.0319 & 1 & 0 \\ -0.6978 & 0.1627 & -0.8602 & 1 \end{bmatrix}.$$
(37)

The standard error of  $B_0$  is

$$\boldsymbol{B}_{0}^{SE} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.2515 & 1 & 0 & 0 \\ 0.2519 & 0.2241 & 1 & 0 \\ 0.2604 & 0.2242 & 0.1259 & 1 \end{bmatrix}.$$
(38)

Eq. (38) shows the most of the standard errors in  $B_0$  are larger than the elements of  $B_0$ . In Table 1, we show the Root Mean Square Error of FR-TVSVAR(1) and SVAR(1).

#### [Table 1 about here.]

In Fig 18 and 19, we show Quantile-Quantile plot and autocorrelation of residuals of the first order of Full Recursive Structural Vector Autoregressions, respectively.

#### [Figure 18 about here.]

#### [Figure 19 about here.]

Second, we estimate SVAR(1) using quarterly data of the Japanese economy from 1980:Q1 to 2006:Q3.  $B_0$  of SVAR(1) is

$$\boldsymbol{B}_{0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0490 & 1 & 0 & 0 \\ -0.4947 & 0.0100 & 1 & 0 \\ -0.7272 & -0.165 & -0.7892 & 1 \end{bmatrix}.$$
(39)

The standard error of  $B_0$  is

$$\boldsymbol{B}_{0}^{SE} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0.2481 & 1 & 0 & 0 \\ 0.2483 & 0.1910 & 1 & 0 \\ 0.2548 & 0.1910 & 0.1157 & 1 \end{vmatrix}.$$
(40)

In Table 2, we show the Root Mean Square Error of FR-TVSVAR(1) and SVAR(1).

In Fig 20 and 21, we show Quantile-Quantile plot and autocorrelation of residuals of the first order of Full Recursive Structural Vector Autoregressions, respectively.

#### [Figure 20 about here.]

#### [Figure 21 about here.]

In Fig. 2, the results of Eq. (33) are shown. Equation (32) represents the monetary policy reaction of the Central Bank of Japan. The call lending rate which is controlled by the Central Bank of Japan assumed to depend on the following four variables, namely, (i) lagged interest rate which represents the smooth adjustment of the interest rate, (ii) rate of inflation, (iii) the growth rate of real GDP  $(y_{t-1})$  and (iv) the exchange rate. Target values of the rate of inflation, log of GDP and the exchange rate are captured in the changes in the constant term  $c_{1,t}$ . Page 20 figures (1) to (5) show the changes in the value of coefficients of Equation (32). Figure 2(1) shows that the central bank of Japan was conducting gradual adjustment of the short term interest rate during 1980 to 1983 when the high rate of inflation in the second oil crisis was observed. However, the Central Bank of Japan stopped to conduct gradual interest rate policy adjustment during the bubble period (1984 to 1990). The Central bank again came back to do gradual interest rate adjustment after 1993 to the recent where low (or zero) interest rate policy was adopted. During the bubble period, the Central bank's call rate control can be seen as abnormal compared with other period. Page 20 Figure (2) shows changes in the coefficient of the lagged rate of inflation ( $\pi_{t-1}$ ). It was positive and quite significant between 1980 to 1982 when the high rate of inflation hit Japan. It denotes that the central bank was watching the rate of inflation as the major target of the monetary policy. However, it turns negative value during the bubble period which suggests the Central Bank of Japan lowered its call lending rate despite low rate of inflation during the period of asset price bubble. Page 20, Figure (3) shows the monetary policy

reaction to real GDP. When Japanese economy faced with sluggish growth, the call lending rate was raised. Since the rate of inflation was the main target of the Central Bank monetary policy in early 1980s. Page 20, Figure (4) denotes the monetary policy reaction to the lagged exchange rate. During the period of early 1980s, despite the yen appreciation the call lending rate was raised in order to fight against higher rate of inflation. Page 20, Figure (5) is the fluctuation of the constant term. In our paper, it suggests the level of the call lending rate. In the period of early 1980s, the level of the call lending rate was high. On the other hand, it had been lowered during the bubble period and early 1990s. The response of the interest rate by the Central Bank of Japan shows no major reactions to any indicators since late 1990s until recent. In Fig. 4, the results of Eq. (35) are shown. Figure 3, (1) is the reaction of the rate of inflation to current nominal interest rate. Higher rate of Inflation raised current nominal interest rate in 198s and early 1990s. Figure 3, (2) is the response to the lagged interest rate of rate of inflation. In early 1980s. tight monetary policy led to lower rate of inflation which is described as negative coefficient of lagged interest rate. Figure 3, (3) is the response to the lagged rate of inflation. During the asset bubble period and early 1990s show negative coefficients. expected rate of inflation, actual rate of inflation was declining. Figure 3, It suggests despite positive (4) is the response of lagged real GDP  $(y_{t-1})$  to the rate of inflation. In mid 1980s, relatively higher growth of the economy brought higher rate of inflation. On the other hand, influence of real GDP to the rate of inflation was quite small. Figure 3, (5) is the response of lagged exchange rate  $(e_{t-1})$  to the rate of inflation. In early 1980s the rate of inflation went on going despite higher appreciation of the yen due to the effect of the second oil crisis. In mid 1980s, high appreciation of the yen brought lower rate of inflation which was one of the causes of the asset price bubble in Japan. Figure 4 shows the aggregate demand function. Figure 4 (3) shows the response of the lagged interest rate on aggregate demand. In 1986, the lower interest rate pushed economic growth. However, in the 1990, lower interest rate kept sluggish economy which is denoted by the positive sign of the lagged interest rate on real GDP. Figure 4 (4) is the response of lagged inflation on aggregate demand. In 1983-85 period, real economy was growing. However, in 1990s, rate of inflation turns to negative figures despite positive low growth rate. Most recent period shows strong positive sign since the rate of inflation and the economic growth turned positive each other. Figure 4 (5) is the response of lagged real GDP on aggregate demand. Figure 4 (6) is the response of lagged exchange rate on aggregate demand. In 1986, despite the appreciation of the yen, real GDP was rising. In 1994 high appreciation of the yen brought slower growth. Figure 5 shows that during the bubble period, inflation had been lowered and at the same time, inflation was stable. Higher economic growth brought appreciation of the yen.

Equation (33) and Figure 3 show the aggregate supply function and their changes in the coefficients. Since it is assumed Cholesky decomposition, the current interest rate  $(i_t)$  appears in Equation (33). The coefficient of it is positive which would be showing the simultaneity of the relation between the rate of inflation  $(\pi_t)$  and the nominal rate of interest  $(i_t)$ . The coefficient of the lagged interest rate  $(i_{t-1})$  is negative during the bubble period of 1985-1990 where lower interest rate induce higher demand and higher rate of inflation. The coefficient of the lagged rate of inflation  $(\pi_{t-1})$  represents the expected rate of inflation. Since 1997, the rate of inflation became almost zero which shows high correlation with the lagged rate of inflation. The coefficient of lagged real GDP  $(y_{t-1})$  shows positive during 1980 to 1987. On the other hand, after 1997, the ordinary Phillips Curve relations can not be observed in Figure 3. (5) in Figure 3 denotes the reaction of the rate of inflation to the exchange rate. When the exchange rate is appreciated, the export will decline and the rate of inflation will fall. Therefore the expected sign of  $e_{t-1}$  is negative. Lastly the constant term (6) increased all the sudden in year 1997 where the target values of the rate of inflation and the real growth rate had been dropped. In Fig. 4, the results of Eq. (34) are shown. The coefficient of  $i_t$  is positive since the real interest rate was rising despite of the decline in nominal interest rate so that the real output did not rise much. From 2001, the growth rate of real output became small so that the coefficient also shows almost zero. The coefficient of  $\pi_{t-1}$  shows positive since the real interest rate becomes lower and the real output rises. The coefficient of  $y_{t-1}$  is negative from 1981 to 1995 since the real growth rate became lower and lower. Especially after 1998 the coefficient show almost zero due to extremely low growth rate. The coefficient of  $e_{t-1}$  shows negative sign most of the period since the appreciation of the yen lowered output growth.

In Fig. 5, the results of Eq. (36) are shown. In Fig. 5, the results of Eq. (35) are shown. The coefficients of interest rates (namely  $i_t$  and  $i_{t-1}$ ),  $b_{4,1,0,t}$  and  $b_{4,1,1,t}$ , show positive sign in the entire period since appreciation of the yen lowers interest rate. The coefficient of inflation rate (namely  $\pi_t$  and  $\pi_{t-1}$ ),  $b_{4,2,0,t}$  and  $b_{4,2,1,t}$ , show positive and negative signs in the entire period. Since the rate of inflation, the real interest rate is lowered. Thus, output increases and capital inflow from abroad. It makes Yen appreciated. The coefficient of inflation rate (namely  $y_t$  and  $y_{t-1}$ ),  $b_{4,3,0,t}$  and  $b_{4,3,1,t}$ , show positive and negative signs in the entire period. Since the increases in output, stock price is expected to rise. Thus, capital inflow increases and the Yen appreciated.

#### 4 Conclusions and Discussions

In this paper, we present empirical analysis on Japanese monetary policy using Time-Varying Structural Vector Autoregressions based on the Monte Carlo particle filter and a self-organizing state space model. We estimate the time-varying reaction function of the Japanese monetary policy, and our TVSVAR also includes an aggregate supply function, an aggregate demand function, and real exchange rate determination function. Our TVSVAR is a dynamic full recursive structural VAR, which is similar to Primiceri (2005), Canova and Gambetti (2006), and many related papers. The most of previous studies are based on Markov Chain Monte Carlo method and the Kalman filter. Our approach, however, is based on the Monte Carlo Particle filter and a self-organizing state space model, proposed by Kitagawa (1996), Gordon et al. (1993), Kitagawa (1998), Yano (2007b), and Yano (2007a). The TVSVAR estimation method is proposed by Yano (2007c). In this paper, we assume on the time evolution of coefficients which is depend on Markov chain. We call this assumption Markov chain priors. The Markov chain priors are the generalization of random walk priors. Thus, the main feature of our method is less restrictions rather than previous methods. Our methods are applied for the estimation of a quarterly model of the Japanese economy (a nominal short term interest rate, the rate of inflation, the growth rate of real output, and the return of the real effective exchange rate). We detect structural changes in most coefficients of TVSVAR. In effectiveness of monetary policy by use of the interest rate can be seen in aggregate supply since 1990. Interest rate policy toward aggregate demand is even worse in a sense that lower interest rate reduced output further. This paper concludes the sluggish economy of Japan is caused not only by aggregate supply factor but also by aggregate demand factor. Ineffectiveness of monetary policy since 1990 could not recover Japanese economy until recent.

For our future study, we would like to try the *p*th order of TVSVAR, estimating time-varying coefficients of exogenous variables, and various type of TVSVAR. Moreover, we would like to try Time-Varying Structural VAR with Stochastic Volatility (TVSVAR-SV) and Time-Varying Structural VAR with Sign Restrictions (TVSVAR-SR). TVSVAR-SV is proposed by Primiceri (2005) and TVSVAR-SR is the dynamic version of Uhlig (2005) <sup>22</sup>.

<sup>&</sup>lt;sup>22</sup>Braun and Shioji (2006) and Kamada and Sugo (2006) analyze the Japanese economy using sign-restricted VAR.

# Appendix A Data Source

We use quarterly macroeconomic data of the Japanese economy from 1980:Q1 to 2006:Q3.

- Uncollateralized overnight call rate, averaged over three months (Bank Of Japan): uncollateralized overnight call rate, monthly average (July 1985 September 2006) and collateralized overnight call rate, monthly average (January 1980 July 1985) are liked at July 1985. http://www.boj.or.jp/en/theme/research/stat/market/index.htm
- Seasonal adjusted real/nominal GDP (Cabinet Office): Quarterly Estimates of GDP, chained, (1994:Q1-2006:Q3) and Quarterly Estimates of GDP, fixed-based, (1980:Q1-1994:Q1) are linked at 1994:Q1. http://www.esri.cao.go.jp/en/sna/menu.html
- Seasonal adjusted GDP deflator (Cabinet Office): deflator is calculated from seasonal adjusted real/nominal GDP.

http://www.esri.cao.go.jp/en/sna/menu.html

• Real effective exchange rate (Bank Of Japan): http://www.boj.or.jp/en/theme/research/stat/market/forex/index.htm

## Appendix B Block Recursive TVSVAR

In subsection 3.1, the macroeconomic variables of FR-TVSVAR(1) are ordered from exogenous variables to endogenous variables. This order of variables is a strong restriction on conventional SVAR. To release the restriction, Christiano et al. (1999) propose "block-recursive" structural VAR. They partition  $Y_t$  into three blocks:

$$\boldsymbol{Y}_t = [\boldsymbol{X}_t, \boldsymbol{M}\boldsymbol{P}_t, \boldsymbol{Z}_t]',$$

where  $X_t$  is a non-monetary block,  $MP_t$  is a monetary policy block,  $Z_t$  is a monetary block.

We estimate the first order of block recursive TVSVAR(BR-TVSVAR(1)) as a benchmark model (from 1980:Q1 to 1998:Q4). BR-TVSVAR(1) is given by

$$y_t = b_{1,1,1,t}y_{t-1} + b_{1,2,1,t}\pi_{t-1} + b_{1,3,1,t}i_{t-1} + b_{1,4,1,t}e_{t-1} + c_{1,t} + \epsilon_{1,t},$$
(B1)

$$\pi_t = b_{2,1,0,t} y_t + b_{2,1,1,t} y_{t-1} + b_{2,2,1,t} \pi_{t-1} + b_{2,3,1,t} i_{t-1} + b_{2,4,1,t} e_{t-1} + c_{2,t} + \epsilon_{2,t}, \tag{B2}$$

$$i_{t} = b_{3,1,0,t}y_{t} + b_{3,2,0,t}\pi_{t} + b_{3,1,1,t}y_{t-1} + b_{3,2,1,t}\pi_{t-1} + b_{3,3,1,t}i_{t-1} + b_{3,4,1,t}e_{t-1} + c_{3,t} + \epsilon_{3,t},$$
(B3)

$$e_{t} = b_{4,1,0,t}y_{t} + b_{4,2,0,t}\pi_{t} + b_{4,3,0,t}i_{t} + b_{4,1,1,t}y_{t-1} + b_{4,2,1,t}\pi_{t-1} + b_{4,3,1,t}i_{t-1} + b_{4,4,1,t}e_{t-1} + c_{4,t} + \epsilon_{4,t}.$$
(B4)

where  $i_t$  is the first difference of the short-term interest rate,  $\pi_t$  is the rate of inflation,  $y_t$  is the growth rate of real output, and  $e_t$  is the return of the real effective exchange rate <sup>23</sup>.

$$b_{x,y,z,t} = sd_{exp}/sd_{obs},$$

 $<sup>^{23}</sup>$ We set the number of particles, M, to 10000. The other parameters of simulation are same in Yano (2007b) and Yano (2007a). All time-varying coefficients are standardized as follows:

where  $sd_{exp}$  is the standard deviation of an explaining variable and  $sd_{exp}$  is the standard deviation of an observation (this standardization method may not be best).

[Figure 22 about here.] [Figure 23 about here.] [Figure 24 about here.]

[Figure 25 about here.]

## Appendix C Non-Gaussian State Space Model

We describe the detail of a non-Gaussian state space model. The time varying coefficients  $b_{i,j,l,t}$  and  $d_{i,n,t}$  are estimated by using MCPF. The non-Gaussian state space representation is given by

$$\boldsymbol{x}_{t} = \boldsymbol{F}\boldsymbol{x}_{t-1} + \boldsymbol{G}\boldsymbol{v}_{t},$$

$$\boldsymbol{y}_{i,t} = \boldsymbol{H}_{t}\boldsymbol{x}_{t} + \boldsymbol{\epsilon}_{i,t},$$

$$i = 1, 2, \cdots, k,$$

$$(C5)$$

where F, G,  $H_t$  are  $(L \times L)$ ,  $(L \times L)$ , and  $(1 \times L)$  matrices, respectively.  $x_t$  is an  $(L \times 1)$  vector of coefficients,  $v_t$  is an L variate possibly non-Gaussian noise,  $\epsilon_{i,t}$  is a possibly non-Gaussian noise, and  $y_{i,t}$  is an observation. The symbol L is kp + n + i - 1. The detail of these vectors and matrices are explained in the following paragraphs. In our algorithm, the matrices F, G are specified as follows.

$$\boldsymbol{F} = \boldsymbol{I}_L, \ \boldsymbol{G} = \boldsymbol{I}_L, \tag{C6}$$

where  $I_L$  is an L-dimensional identity matrix.

For the convenience of the expression, we use the following notations:

The vectors,  $x_t$  and  $H_t$  are defined as follows. For the first component of  $y_t$ , i = 1,

$$\begin{aligned} \boldsymbol{x}_t &= \left(\hat{\boldsymbol{b}}_{1,t}, \hat{\boldsymbol{d}}_{1,t}\right)^T, \\ \boldsymbol{H}_t &= \left(\hat{\boldsymbol{h}}_t, \hat{\boldsymbol{f}}_t\right). \end{aligned} \tag{C8}$$

For the *i*th component of y(t),  $1 < i \le k$ ,

$$\begin{aligned} \boldsymbol{x}_t &= \left( \hat{\boldsymbol{b}}_{i,0,t}, \hat{\boldsymbol{b}}_{i,t}, \hat{\boldsymbol{d}}_{i,t} \right)^T, \\ \boldsymbol{H}_t &= \left( \hat{\boldsymbol{r}}_{i,t}, \hat{\boldsymbol{h}}_t, \hat{\boldsymbol{f}}_t \right). \end{aligned} \tag{C9}$$

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Figure 1: Macroeconomics Data of Japan: 1980:I-2006:IV





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Figure 4:  $y_t = b_{3,1,0,t}i_t + b_{3,2,0,t}\pi_t + b_{3,1,1,t}i_{t-1} + b_{3,2,1,t}\pi_{t-1} + b_{3,3,1,t}y_{t-1} + b_{3,4,1,t}e_{t-1} + c_{3,t} + \epsilon_{3,t}$ 



Figure 5:  $e_t = b_{4,1,0,t}i_t + b_{4,2,0,t}\pi_t + b_{4,3,0,t}y_t + b_{4,1,1,t}i_{t-1} + b_{4,2,1,t}\pi_{t-1} + b_{4,3,1,t}y_{t-1} + b_{4,4,1,t}e_{t-1} + c_{4,t} + \epsilon_{4,t}$ 



Figure 6: FR-TVSVAR(1): Impulse Response (1980:Q4)



Figure 7: FR-TVSVAR(1): Impulse Response (1985:Q4)



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Figure 9: FR-TVSVAR(1): Impulse Response (1997:Q4)



Figure 10: Q-Q Plot (Full Recursive TVSVAR)



Figure 11: Autocorrelation with 95% Confidence Interval (Full Recursive TVSVAR)



Figure 12:  $i_t = b_{1,1,1,t}i_{t-1} + b_{1,2,1,t}\pi_{t-1} + b_{1,3,1,t}y_{t-1} + b_{1,4,1,t}e_{t-1} + c_{1,t} + \epsilon_{1,t}$ 



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![](_page_35_Figure_0.jpeg)

Figure 16: Q-Q Plot (Full Recursive TVSVAR)

![](_page_36_Figure_0.jpeg)

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Time

Figure 22:  $y_t = b_{1,1,1,t}y_{t-1} + b_{1,2,1,t}\pi_{t-1} + b_{1,3,1,t}i_{t-1} + b_{1,4,1,t}e_{t-1} + c_{1,t} + \epsilon_{1,t}$ 

![](_page_42_Figure_0.jpeg)

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![](_page_43_Figure_0.jpeg)

Figure 24:  $i_t = b_{3,1,0,t}y_t + b_{3,2,0,t}\pi_t + b_{3,1,1,t}y_{t-1} + b_{3,2,1,t}\pi_{t-1} + b_{3,3,1,t}i_{t-1} + b_{3,4,1,t}e_{t-1} + c_{3,t} + \epsilon_{3,t}$ 

![](_page_44_Figure_0.jpeg)

Figure 25:  $e_t = b_{4,1,0,t}y_t + b_{4,2,0,t}\pi_t + b_{4,3,0,t}i_t + b_{4,1,1,t}y_{t-1} + b_{4,2,1,t}\pi_{t-1} + b_{4,3,1,t}i_{t-1} + b_{4,4,1,t}e_{t-1} + c_{4,t} + \epsilon_{4,t}$ 

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2	Root Mean Square Error	•		•		•	•		•	•	 •	•	•	•		•		•			4	7

RMSE	Time-Varying SVAR	Fixed Coefficient SVAR
$\epsilon_i$	0.01950	0.44802
$\epsilon_{\pi}$	0.10686	0.50403
$\epsilon_y$	0.04861	0.89696
$\epsilon_e$	0.24998	4.15722

Table 1: Root Mean Square Error

RMSE	Time-Varying SVAR	Fixed Coefficient SVAR
$\epsilon_i$	0.04303	0.45066
$\epsilon_{\pi}$	0.11014	0.52432
$\epsilon_y$	0.08479	0.89801
$\epsilon_e$	0.08479	4.15124

Table 2: Root Mean Square Error